LESSON 11

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

SIGNS OF SHAPE

11.1 Definition: A sign of shape is a miniature picture of a geometric figure or an object.

△ (triangle) ∠ (angle) ○ (circle)

Basic Shapes

A basic shape is represented in braille by the shape indicator followed by a numeral, one or more letters, or a dot combination suggestive of the shape.

Shape Indicator	⠫

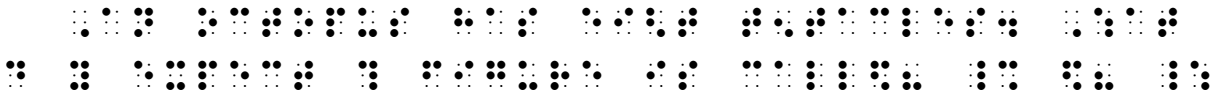
11.2 Basic Signs of Shape Represented by Numbers—Regular Polygons: A closed figure that has equal sides and equal angles is called a *regular polygon* and is represented by the shape indicator followed by a numeral specifying the number of sides in the figure.

Square (4-sided)	□	⠫⠙
Regular Pentagon (5-sided)	⬠	⠫⠑
Regular Hexagon (6-sided)	⬡	⠫⠋

*Note that the equilateral triangle, which is a regular polygon, is not represented by the number three. See **11.4**.*

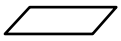
11.2.1 Unlisted Regular Polygons: Symbols which represent regular polygons with seven or more sides are not provided for in the Nemeth Code. If the unlisted shape is a *regular polygon*—that is, it is a closed figure with equal sides and equal angles—the transcriber is instructed to devise a symbol in accordance with the principles above, based on the number of sides the shape has. It may be helpful to include a tactile drawing of the shape. Unlisted regular polygon constructions do not require a transcriber's note.

Example 11.2-1 An octopus has eight tentacles. What do you expect this figure is called?



In addition to the transcriber-devised braille symbol, the shape is presented as a tactile graphic at the first mention of this shape. Follow directives in the most recent edition of Guidelines and Standards for Tactile Graphics.

11.3 Basic Signs of Shape Represented by Letters—Irregular Polygons: A closed figure which has at least two unequal sides and/or two unequal angles is called an *irregular polygon* and is represented by the shape indicator followed by a letter or a combination of letters suggestive of the name of the shape. (The derivation of the letter following the shape indicator is underlined in the list below.)

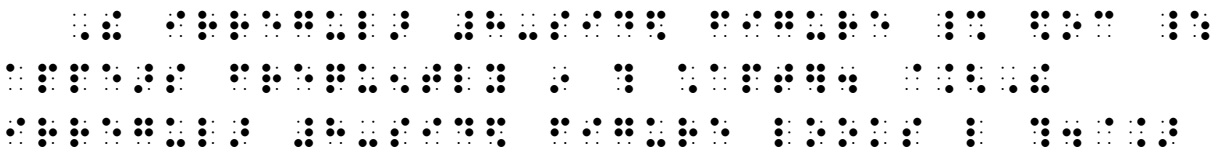
Diamond		
Irregular Hexagon		
Irregular Pentagon		
Parallelogram		
Quadrilateral		
Rectangle		
Rhombus		
Trapezoid		

We apologize for the blurry images in this lesson. We hope to provide better graphics in future editions.

11.3.1 Unlisted Irregular Polygons: You may come across an irregular polygon which is not provided for in the Nemeth Code. If the unlisted shape is an *irregular polygon*—that is, it is a closed figure with at least two unequal sides and/or two unequal angles—the transcriber is instructed to devise a symbol if it appears frequently in the transcription. Construct the symbol in accordance with the principles above. Be careful not to choose a letter or letter combination which already has an assigned meaning in the Nemeth Code. Refer to Appendix B of the Nemeth Code for a list of symbols already in use. Symbols beginning with dots 1246 begin on page 222.

A transcriber's note is required to define the figure unless it is described in the narrative. In your note, give the name or description of the symbol used. Include a drawing of the shape when appropriate.

Example 11.3-1 The irregular 8-sided figure  appears frequently in this chapter.



A raised outline of the shape is included as a tactile graphic at the first mention of this shape. Follow directives in the most recent edition of Guidelines and Standards for Tactile Graphics.

11.4 Other Basic Signs of Shape Represented by Letters: The following shapes are also represented by the shape indicator followed by a letter suggestive of the name. (The derivation of the letter following the shape indicator is underlined.)

Circle	○	
Ellipse	⬭	
Intersecting Lines	×	
Star	☆	
Triangle	△	

Clarification: The triangle shape shown here is an equilateral triangle – sides are equal in length, angles are equal in measurement. The braille symbol  is used to represent a triangle shape when the print copy uses the drawing to replace the word "triangle" or when it is used to specifically represent an equilateral triangle. In the study of triangles, other types of triangles named. These are discussed in **11.7**.

The following shapes were introduced in **Lesson 6** as signs of comparison. They may also be used in print to simply replace the word they represent. Notice that two signs in this category begin with the negation symbol (34) immediately followed by the shape indicator.

Arc, Concave Upward	⌒	⠸⠩
Is Parallel To	∥	⠸⠳
Is Not Parallel To	∦	⠰⠸⠳
Is Perpendicular To	⊥	⠸⠾
Is Not Perpendicular To	⊥̸	⠰⠸⠾

Example 11.4-1 Line AD ∦ Line BC.

⠸⠁ ⠰⠸⠳ ⠠⠁⠙ ⠰⠸⠳ ⠠⠃⠉

11.4.1 Other Unlisted Basic Shapes: Basic shapes not provided for in the Nemeth Code are formed in accordance with the principles above. One must be careful not to choose a symbol which already has an assigned meaning in the Nemeth Code. Refer to Appendix B of the Nemeth Code for a list of symbols already in use. Symbols beginning with dots 1246 begin on page 222; symbols beginning with dots 34 are on page 231.

A transcriber's note is required to define the figure. Give the name or description of the symbol used. Include a drawing of the shape if it is vital to the mathematical topic at hand.

Example 11.4-2 VALENTINE MATH: Replace the heart with the correct math symbol: $14 \heartsuit 2 = 7$.

⠠⠠⠠⠍⠁⠞⠓ ⠎⠽⠍⠃⠕⠇⠎
⠠⠠⠠⠗⠑⠏⠇⠁⠉⠑ ⠮ ⠓⠑⠁⠗⠞ ⠺⠊⠹ ⠞⠓⠑ ⠉⠕⠗⠗⠑⠉⠞
⠍⠁⠞⠓ ⠎⠽⠍⠃⠕⠇⠲
⠠⠠⠠⠝⠕⠞⠑ ⠠⠗⠑⠋⠑⠗ ⠞⠕ ⠞⠓⠑ ⠍⠕⠎⠞ ⠗⠑⠉⠑⠝⠞ ⠠⠁⠉⠑⠍⠑ ⠠⠛⠥⠊⠙⠑⠇⠊⠝⠑⠎
⠁⠝⠙ ⠠⠎⠞⠁⠝⠙⠁⠗⠙⠎ ⠋⠕⠗ ⠞⠁⠉⠞⠊⠇⠑ ⠛⠗⠁⠏⠓⠊⠉⠎ ⠁⠝⠙ ⠞⠓⠊⠗⠙ ⠛⠗⠁⠙⠑
⠍⠁⠞⠻⠊⠁⠇⠎ ⠊⠝ ⠅⠊⠝⠙⠑⠗⠛⠁⠗⠞⠑⠝ ⠞⠓⠗⠕⠥⠛⠓ ⠞⠓⠊⠗⠙ ⠛⠗⠁⠙⠑⠲

The transcriber represents the heart shape with ⠸⠩⠳ since ⠸⠳ means "rhombus".

Note: Refer to the most recent edition of *Guidelines and Standards for Tactile Graphics* regarding picture objects (such as counting symbols, pictographs, etc.) used in Kindergarten through third grade materials.

11.5 Basic Signs of Shape Represented by Other Dot Combinations: Three additional shapes are identified in the Nemeth Code.

Angle	∠	
Arc, Concave Downward	◡	
Inverted Triangle	▽	

11.6 Filled-In and Shaded Shapes: A filled-in or shaded closed shape (circle, diamond, square, etc.) is represented as such by the filled-in shape indicator or the shaded shape indicator. The appropriate indicator is placed between the shape indicator and the shape symbol.

Filled-in shape indicator	
Shaded shape indicator	

 (filled-in star)

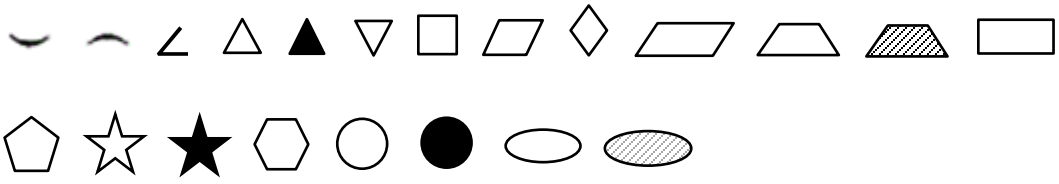
 (shaded circle)

Shapes used as icons in non-mathematical context are discussed later in this lesson. See **11.26**.

Instructions: Leave one space between each shape. Braille as many shapes on one braille line that will fit before beginning a new line. Use "fl" to represent the flower shape and "ch" to represent the chicken. The required transcriber's note may be omitted in this practice exercise.

PRACTICE 11A

Listed Shapes



Unlisted Shapes

Shapes with Structural Modification

11.7 Definition and Construction: A shape with structural modification is one in which the general print form of a basic shape (such as *triangle*) is changed to show a more specific form (such as *right triangle*).

Basic shape: Triangle 

More specific form: Right Triangle 

Composite signs in which two or more signs of shape are combined are also structurally modified shapes, for example, two *angle* shapes in print combine to form the symbol for *adjacent angles*.

Basic shape: Angle ∠

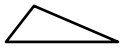
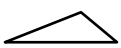
More specific form: Adjacent Angles 

A shape with structural modification is represented by

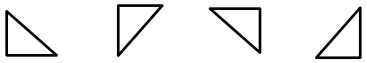
- the basic shape symbol,
- followed by the structural shape-modification indicator,
- followed by a letter or an uncontracted combination of letters suggestive of the change in the shape,
- ending with the termination indicator which signals the end of the modification.

Shape indicator	⠸⠩
Structural shape-modification indicator	⠐⠣
Termination indicator	⠸⠱

11.8 Structurally Modified Triangles: The following five structurally modified triangles are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "triangle" ⠸⠩. The derivation of the letter following the structural shape-modification indicator ⠐⠣ is underlined in the list of modified triangles below.

<u>A</u>cute Triangle		⠸⠩⠐⠣⠁⠸⠱
<u>I</u>sosceles Triangle		⠸⠩⠐⠣⠊⠸⠱
<u>O</u>btuse Triangle		⠸⠩⠐⠣⠕⠸⠱
<u>R</u>ight Triangle		⠸⠩⠐⠣⠗⠸⠱
<u>S</u>calene Triangle		⠸⠩⠐⠣⠎⠸⠱

Know Your Triangles: Triangles are defined by the measure of angles and sides, not by orientation. For example, each of these is a "right triangle" because each contains a right (90°) angle.



*Definitions can be found in **Appendix B** of this course ("Glossary of Terms").*

11.9 Structurally Modified Angles: The following twelve structurally modified angles are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "angle" ⠸⠐. The derivation of the letter or letters following the structural shape-modification indicator ⠸ is underlined in the list below.

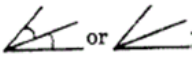
Specific Angles

Obtuse Angle  ⠸⠐⠕⠝

Right Angle  ⠸⠐⠗⠁

Straight Angle ⠸⠐⠎⠞

Combined Angles

Adjacent Angles  ⠸⠐⠁⠙

Alternate Exterior Angles  ⠸⠐⠁⠑

Alternate Interior Angles  ⠸⠐⠁⠊

Complementary Angles  ⠸⠐⠉⠁

Corresponding Angles  ⠸⠐⠉⠕

Exterior Angles  ⠸⠐⠑⠭

Interior Angles  ⠸⠐⠊⠝

Supplementary Angles  ⠸⠐⠎⠥

Vertical Angles  ⠸⠐⠧⠑

11.10 Unlisted Shapes with Structural Modification: Structurally modified shapes which are not provided for in the Nemeth Code are formed in accordance with the principles above. Review the definition of *structural modification* in **11.7** to properly identify the unlisted shape. Be careful not to choose a symbol which already has an assigned meaning in the Nemeth Code. Refer to Appendix B of the Nemeth Code for a list of symbols already in use. Symbols beginning with dots 1246 begin on page 222.

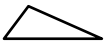
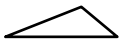
Explain the unlisted shape in a transcriber's note, giving the name or description of the symbol used. Include a drawing of the shape when appropriate.

Instructions: Review simple table format in **Lesson 6**. Do not braille tables side-by-side. After completing the "Angle/Symbol" table, leave one blank line and then begin the "Triangle/Symbol" table. Do not use box lines.

PRACTICE 11B

Structurally Modified Shapes

Angle	Symbol
right	
straight	
obtuse	
complementary	
supplementary	
vertical	

Triangle	Symbol
isosceles	
right	
acute	
obtuse	
scalene	

Shapes with Interior Modification

11.11 Definition and Construction: A shape with interior modification is a basic shape (for example, a *circle*) within which a letter, a numeral, a sign of operation, or other sign appears.

Basic shape: Circle ○

More specific form: Circle with number 8 inside ⑧

More specific form: Circle with asterisk inside ⊛

A shape with interior modification is represented by

- the basic shape symbol,
- followed by the interior shape-modification indicator,
- followed by the symbol corresponding to the interior sign,
- ending with the termination indicator which signals the end of the modification.

Shape indicator	⠸
Interior shape-modification indicator	⠐⠸
Termination indicator	⠸

Note: Symbols, numbers, words, etc. that represent keys on a calculator or a keyboard follow rules for keystrokes. See ***11.23****.*

11.12 Circles with Interior Modification: Eleven circles with interior modification are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "circle" ⠸⠴ followed by the interior shape-modification indicator ⠐⠸ . Notice that an interior numeral includes a numeric indicator and that the contracted form of the right-pointing arrow is not used.

Circle with Interior Arrow Pointing Right	⊕	⠸⠴⠐⠸⠳⠀⠀⠀⠀⠀⠀
Circle with Interior Arrow Pointing Left	⊕	⠸⠴⠐⠸⠪⠀⠀⠀⠀⠀⠀
Circle with Interior Arrow Pointing Up	⊕	⠸⠴⠐⠸⠕⠀⠀⠀⠀⠀⠀
Circle with Interior Arrow Pointing Down	⊕	⠸⠴⠐⠸⠇⠀⠀⠀⠀⠀⠀
Circle with Interior Capitalized Letter	Ⓦ	⠸⠴⠐⠸⠠⠺
Circle with Interior Numeral	⑤	⠸⠴⠐⠸⠼⠑
Circle with Interior Cross	⊗	⠸⠴⠐⠸⠭

Circle with Interior Dot 

Circle with Interior Minus Sign 

Circle with Interior Plus Sign 

Circle with Interior Vertical Bar 

11.13 Angles with Interior Modification: Three angles with interior modification are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "angle" ⠸⠐ followed by the interior shape-modification indicator ⠸⠸

Angle with Interior Arc 

Angle with Interior Clockwise Arrow 

Angle with Interior Counterclockwise Arrow 

Note: When the print copy uses the "angle with interior arc" symbol throughout the text to simply mean "angle", the simple braille shape symbol for "angle" ⠸⠐ may be used. A transcriber's note is required to inform the reader of the substitution.

11.14 Rectangles and Squares with Interior Modification: One rectangle and seven squares with interior modification are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "rectangle" ⠸⠶ or for "square" ⠸⠒ followed by the interior shape-modification indicator ⠸⠸

Rectangle with Interior Horizontal Bar 

Square with Interior Bar

Interior Horizontal Bar 

Interior Vertical Bar 

Square with Interior Diagonal

from Lower-Left to Upper-Right 

from Upper-Left to Lower-Right 

Square with Interior Diagonals 

Square with Interior Dot 

Square with Interior Numeral 

11.15 Words Enclosed in Shapes: Words enclosed in shapes are transcribed according to the methods for shapes with internal modification and must be enclosed within Nemeth switches. *Note: Words that represent keys on a calculator or a keyboard follow rules for keystrokes. See* ***11.23****.*

11.16 Two or More Vertically Arranged Modifiers: When two or more vertically arranged symbols occur within a basic sign of shape, the basic shape symbol and the interior shape-modification indicator are followed first by the symbol for the upper and then by the symbol for the lower interior sign. The termination indicator is used only after the last symbol.

Circle with Interior Arrow Pointing Right

Over Interior Arrow Pointing Left ..... 

Circle with Interior Arrow Pointing Left

Over Interior Arrow Pointing Right .... 

11.17 Two or More Horizontally Arranged Modifiers: When two or more horizontally arranged symbols occur within a basic sign of shape, a multipurpose indicator (dot 5) is inserted between the interior modifiers to show that they are printed horizontally, not vertically. The termination indicator is used only after the last symbol.

Circle with Interior Arrow Pointing Up

Followed by Interior Arrow

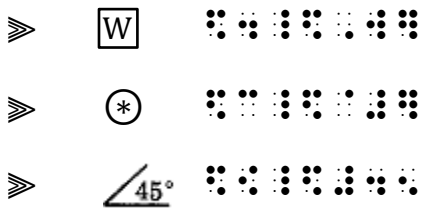
Pointing Down ..... 

Circle with Interior Arrow Pointing Down

Followed by Interior Arrow

Pointing Up ..... 

11.18 Unlisted Shapes with Interior Modification: Shapes with interior modification not provided for in the Nemeth Code are formed in accordance with the principles for the construction of such shapes. Review the definition of *interior modification* in **11.11** to properly identify the unlisted shape.



⋙ ⠷ ⠐⠣⠐⠐⠣⠐⠣⠐⠐⠜

A symbol which already has an assigned meaning in the Nemeth Code must not be used for the unlisted sign of shape. If necessary, explain the shape in a transcriber's note giving the name or description of the symbol used. Include a drawing of the shape when appropriate.

Instructions: Braille this as a simple list, not as columns.

PRACTICE 11C

Squares with Interior Modification

Square with interior numeral 2 ⠷⠼⠃⠾

Square with interior dot ⠷⠐⠄⠾

Square with interior horizontal bar ⠷⠤⠾

Square with interior vertical bar ⠷⠳⠾

Square with interior diagonals ⠷⠭⠾

Other Details

11.19 Spacing with Signs of Shape: Except for keystroke constructions (see **11.24.2** below), a sign of shape is spaced in accordance with its assigned meaning. For example, operation signs are unspaced,

⋙ $x \oplus y$ ⠷⠭⠐⠣⠽⠾

and comparison signs are spaced.

⋙ $x \ominus y$ ⠷⠭⠐⠠⠣⠽⠾

Example 11.19-1 Operation signs appear within circles. The comparison sign appears inside a square.

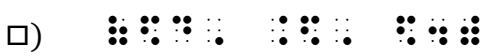
$2 \oplus 3 \boxed{=} 3 \oplus 2$

⠼⠃⠐⠣⠖⠐⠜⠼⠉ ⠷⠨⠅⠾ ⠼⠉⠐⠣⠖⠐⠜⠼⠃ ⠄⠄⠄ ⠼⠃⠐⠣⠖⠐⠜⠼⠉ ⠷⠨⠅⠾ ⠼⠉⠐⠣⠖⠐⠜⠼⠃

⠄⠄⠄ ⠼⠃⠐⠣⠖⠐⠜⠼⠉ ⠷⠨⠅⠾ ⠼⠉⠐⠣⠖⠐⠜⠼⠃

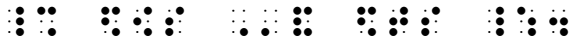
11.20 Punctuation with Signs of Shape: Signs of shape are punctuated mathematically when the punctuation falls within the code switches.

⋙ (◊, ▽, □)



11.21 Plurals/Possessives: The uncapitalized letter "s" or the apostrophe-s combination occurring *inside or after* a sign of shape to show its plural or possessive are placed <u>after</u> the shape symbol in braille. Apply the general rules for the English letter indicator to the plural or possessive ending.

Example 11.21-1 ∠s and △s.



Each "s" is printed inside the shape.

Example 11.21-2 ∠s and △s.

Each "s" follows the printed shape.

Example 11.21-3 ∠'s and △'s.

A punctuation indicator precedes each apostrophe.

Example 11.21-4 (∠'s, △'s, and ◯'s.)

Each "apostrophe-s" is punctuated mathematically because each is associated with a mathematical item.

11.22 Further Considerations Regarding Transcriber-Devised Shapes: As previously noted, when encountering a shape not provided for in the Nemeth Code the transcriber may devise a symbol if that shape appears more than occasionally. The print shape should also be drawn as a raised-line diagram the first time the new symbol is introduced. In addition to the guidelines regarding unlisted shapes throughout this lesson, observe the following.

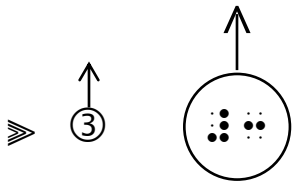
11.22.1 Usage Rules Regarding Interior Numerals and Arrows: Transcriber-devised forms should heed the following principles regarding interior numerals and arrows.

— The numeric indicator is used before a *numeral* or before *a decimal point and a numeral* following the interior shape-modification indicator.

⋙ ⑧

— If a *right-pointing arrow* in regular type with a full barb and single shaft of ordinary length is part of a shape symbol, its contracted form is not used.

11.22.2 Shapes Represented by Drawing: Drawn-in shapes are often more readable than elaborate braille constructions. Since it is not possible to formulate specific rules for the selection of an appropriate form, the decision is left to the experience and judgment of the transcriber. Shapes may also be represented by a combination of drawing and braille symbols. For example, if a modified shape cannot be represented clearly by braille symbols alone, the shape can be drawn and the modification shown in braille.



PRACTICE 11D

1. ▯, ◯, △, ∠, ◎, ∠ .
2. (◯'s, ∠'s, and △'s.)
3. a ⊕ (b ⊕ c)
4. r ⊗ s ⊗ ____ = rst
5. How many Ⓢ△ can you find in the giant ▢ ?

11.23 The Keystroke Indicator: When a print shape with interior modification depicts a labeled calculator or computer key, a contracted form employing a keystroke indicator is used in braille. A keystroke is represented by

— the keystroke indicator,
— followed by the label printed on the calculator key or the computer key,
— ending with the termination indicator which signals the end of the modification.

Keystroke indicator	⠸⠡
Termination indicator	⠸⠱

11.23.1 Shape in Print: The keystroke indicator is used regardless of the shape of the key in the print copy.

⠸⠡ [+] ⠸⠡⠢⠖

⠸⠡ ⊛ ⠸⠡⠔⠢⠖

The actual key shape(s) used in a particular text should be specified on the Transcriber's Notes page. For example, "Calculator keys are depicted in print as square shapes."

11.24 Other Details Concerning Keystrokes

11.24.1 The Label: Regarding the item depicted on the key, note the following.

— Capitalization is duplicated in braille.

⠸⠡ [ENTER] ⠸⠡⠠⠠⠠⠑⠝⠞⠑⠗

⠸⠡ [Enter] ⠸⠡⠠⠑⠝⠞⠑⠗

Reminder: Words are brailled without contractions.

— Follow the usual rules of the Nemeth Code for typeform and use of indicators.

⠸⠡ [x^2] ⠸⠡⠭⠰⠼⠃

Italic typeform for variables is disregarded. A baseline indicator is required before brailling the termination indicator in this example because the keystroke is on the baseline of writing.

Example 11.24-1 Press [F1] for help.

*A multipurpose indicator (dot 5) is needed to show that the numeral is not a subscript. Review section **7.16**.*

— The numeric indicator is not required within the keystroke construction.

» [8]

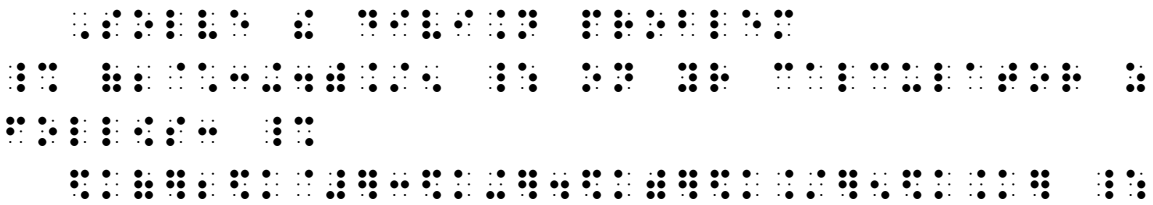
*Compare to a shape with interior modification which <u>does</u> require a numeric indicator. See **11.22.1**.*

11.24.2 Spacing

— No space is left between keystroke constructions and other similar constructions or mathematical symbols in a sequence of related calculations.

Example 11.24-2 Solve the division problem $(2 \times 3 + 4) \div 5$ on your calculator as follows:

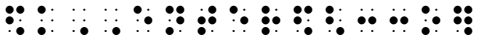
[(] 2 [*] 3 [+] 4 [)] [÷] 5 [=]



— Arrows contained in the labels on the keys should not be spaced from the material to which they apply.

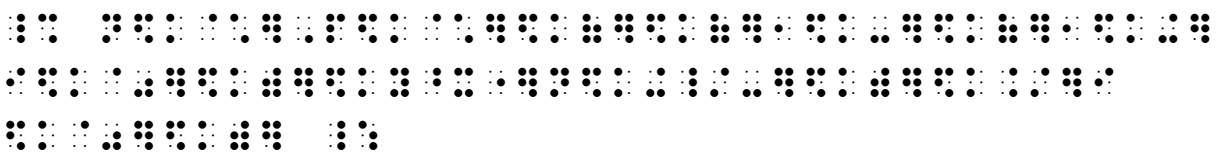
» [$A^b/_c \leftrightarrow {}^d/_e$]

» [**ENTER ↑**]



11.25 Long Keystroke Constructions: Use the fullest extent of the current braille line, making sure that a single keystroke construction is not divided between braille lines. (When the print lines are arranged in a logical sequence, duplicate the arrangement of the print lines if it is possible.)

Example 11.25-1 n ⊗ P ⊗ ((1 − (1 + i %) y^x n +/−) ÷ i %)



Icons

11.26 Consistency in Representation of Icons: When non-word UEB symbols such as icons appear in mathematical context, the symbol may be brailled in Nemeth context using the UEB transcriber-defined shape indicator ⠸⠶ This UEB indicator may be transcribed in either UEB context or Nemeth context without the insertion of switch indicators. List the icon on the Special Symbols page.

Example 11.26-1 (Earlier in the book it is stated that a gold ribbon indicates extra credit problems.)

Evaluate each expression.

13. $\frac{1}{2}(5 + 13) - 4 \cdot 5$

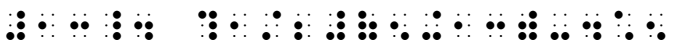
14. $(5 + 11) - (24 - 15) \cdot (3)$

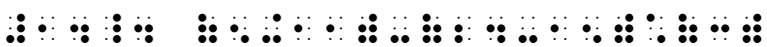
15. $6^2 + 3 \cdot 7 - 9 \div 3$

The BANA Nemeth Code Technical Committee is discussing details regarding alignment of identifiers when some items are marked. The transcription of this example will be revisited after rulings are made.

PROVISIONAL SOLUTION

1 

2 

3 

4 

Line 1: Instructions begin in cell 5. The opening Nemeth Code indicator is placed at the end of this line of text so that all identifiers may start in the same cell.

Line 4: An icon was created to represent the "extra credit" print symbol. There is no need to switch out of Nemeth Code in order to braille the icon. The icon is listed on the Special Symbols page as a UEB symbol as follows.

Shapes Used as Signs of Omission

11.27 Spacing: When a sign of shape is used as a sign of omission or placeholder to represent a numeral, letter, sign of comparison, sign of operation, abbreviation, or any other item, the sign of shape is spaced in accordance with the rules for the omitted material it represents.

⋙ $12 \div 4 = \square$ *(a numeral is omitted—symbol is spaced from comparison sign)*

⋙ 40 dimes = $ ◯ *(a numeral is omitted— symbol is unspaced from monetary symbol)*

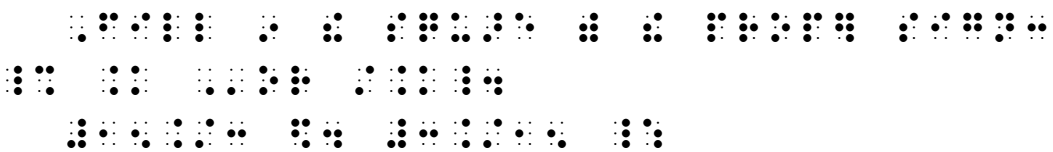
⋙ □ l = 1000 cc *(a numeral is omitted— symbol is spaced from abbreviation "l")*

⋙ 15 ◯ 15 = 30 *(an operation sign is omitted— symbol is unspaced)*

⋙ 24 hrs. = 1 △ *(an abbreviation is omitted— symbol is spaced from preceding numeral)*

Example 11.27-1 Fill in the square with the proper sign: $=$ or $\neq$.

$15 \div 3 \square 3 \div 15$



A sign of shape is unspaced from any braille indicator which applies to it.

⋙ $24_{\Diamond} + 11_{\Diamond} = 40_5$ *(numerals in subscript position are omitted)*

⋙ $y^{\frac{1}{3}} = \sqrt[3]{\blacksquare}$ *(radicand is omitted)*

11.28 The English Letter Indicator and Comparison Signs: When a sign of shape is used in place of a comparison sign, rules regarding use of the English letter indicator to items immediately preceding or following the shape apply just as they would for any comparison sign. In these three examples, a "square" symbol is used as a sign of omission representing a comparison sign.

⋙ x □ y

Rule: An ELI is not used for a single letter which is immediately preceded or followed by a sign of comparison.

⋙ 1 yr ◯ 200 da

Rule: An ELI is needed for an abbreviation that corresponds to a shortform when the abbreviation has no period.

⋙ ray ab ⬭ ray ac

Rules: "ab" – An ELI is not used for a letter combination corresponding to a shortform when it is immediately followed by a comparison sign. "ac" – An ELI is needed for a lowercase letter combination that corresponds to a shortform that is preceded by a space if it is not immediately next to a sign of comparison.

⋙ iii □ iv

Rule: An ELI is not used for a Roman numeral which is immediately preceded or followed by a sign of comparison.

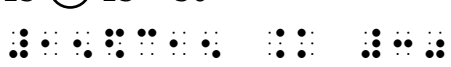
11.29 Use of the Multipurpose Indicator: Because a regular polygon with four or more sides is represented by a numeral, when an unspaced numeral follows such a shape, a multipurpose indicator is used before the numeral to prevent it from being misread. The shapes in these two examples represent a sign of operation.

⋙ 15 □ 15 = 30

Without the multipurpose indicator this is read as a 415-sided regular polygon.

The situation stated above occurs only when the shape is represented by a numeral.

⋙ 15 ◯ 15 = 30

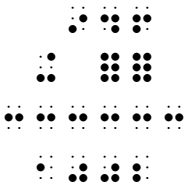


The circle shape is represented in braille with a letter ("c"). The following numeral will not be misread.

11.30 Omissions in Spatially-Arranged Problems: In a spatial arrangement, omissions are indicated with the general omission indicator regardless of the symbol used in print.

Example 11.30-1

```
   9 4 6
+  □□
-------
1 0 0 2
```



In print, the omissions are indicated as two squares.

Instructions: Use the word "pencil" to name the icon shown below. After the completion of this practice drill, show how the icon will be listed on the Special Symbols page.

PRACTICE 11E

(2 * 3 + 9) ÷ 5 =

9 · 35 y^x · 17 +/– =

2 1 2 ° → °C End

1. Fill in the box with the correct exponent.

   a. $x^2 \times x^4 = x^{\square}$

   b. $y^3 \times y^{\square} = y^9$

   c. $z^{\square} \times z^5 = z^{15}$

Show your work with problems marked with ✎ .

A. $436 - \bigcirc = 102$

B. $5_8 + \bigcirc_8 = 22_8$

✎ C. $5\frac{18}{12} = \square\frac{1}{2}$

D. Name two different operation signs that make this a true statement.

$1 \square 1 = 1$

✎ E. $\frac{15}{20} = \frac{3}{\square}$

Identified Signs of Shape

A sign of shape which is followed by a letter, a sequence of letters, or a numeral, is an *identified sign of shape.* The entire unit is mathematical and therefore brailled in Nemeth Code.

11.31 Spacing: There must be a space between the shape symbol and its identification. A space often does not appear in the print copy but it must be present in braille.

⋙ △ABC ("triangle ABC")

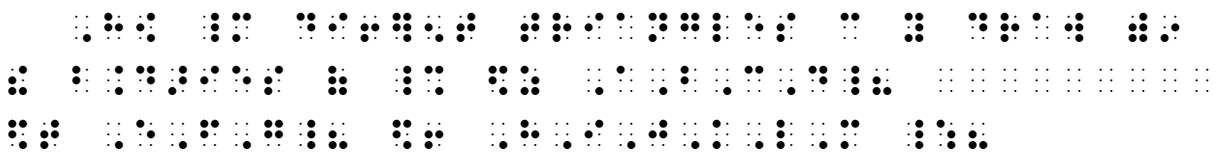
⋙ ∠θ ("angle theta")

Example 11.31-1 ⏢5 denotes "trapezoid 5."

The first 5 is associated with the mathematical shape and is included inside the switches. A switch to Nemeth Code is not necessary for the second 5 – it is a freestanding numeral in UEB context.

11.31.1 Keep Together: A sign of shape and the letter, sequence of letters, or numeral which follows it is regarded as a single mathematical item and therefore should not be divided between braille lines.

Example 11.31-2 How many different triangles can you draw within the boundaries of ⏢ABCD? △EFG? ⎔HIJKLM?

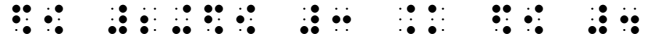


11.31.2 Surrounding Symbols: The spacing before and after a sign of shape and its identification is subject to the spacing rules for the symbols preceding or following it.

⋙ △PQR ~ △P'Q'R'

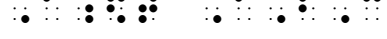
A space precedes and follows the tilde, which, in this case, is a sign of operation meaning "is similar to".

⋙ ∠2 + ∠3 = ∠4

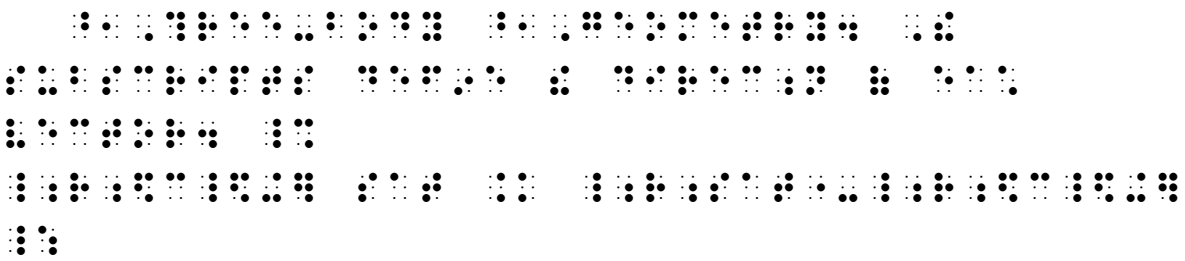


There is no space before or after the operation sign (plus sign).

11.32 A Shape Within a Superscript or a Subscript: An identified sign of shape is read as a unit, therefore when one appears in a superscript or a subscript the effect of the level indicator extends through the space following the sign of shape. That is, the space preserves the superscript or subscript level where the sign of shape appears.

⋙ $A_{\triangle ABC}$ 

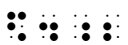
Example 11.32-1 **Three-body Geometry.** The subscripts define the direction of each vector. $\mathbf{r}_{\oplus sat} = \mathbf{r}_{sat} - \mathbf{r}_{\oplus}$



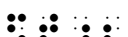
"⊕sat" is in the subscript position. Only one subscript indicator is needed because the level continues through the space following the shape. (Reminder: Boldface type used to identify letters as vectors must be preserved.)

11.33 A Shape Which Carries a Superscript or a Subscript: When a sign of shape carries a superscript or subscript, the level indicator is unspaced from the shape.

⋙ $\triangle^2$ 

⋙ $\square_2$ 

The rule for nonuse of the subscript level indicator (see **8.10**) does not apply to a shape represented by a letter. A subscript indicator is required.

⋙ $\triangle_2$ 

With an identified sign of shape, the space required between the sign of shape and its identifier follows the superscript or subscript. When the identified shape is on the baseline of writing, the space following the superscript or subscript terminates the effect of the level indicator and reinstates the baseline level.

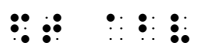
⋙ $\triangle^2$ DEF 

Only the numeral "2" is in the superscript position. The shape and letters "DEF" are on the baseline of writing.

11.34 The English Letter Indicator: When an English letter, a Roman numeral, or a shortform letter combination identifies a shape, the English letter indicator is not used.

⋙ $\angle p$  ("angle p")

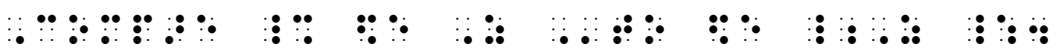
⋙ □ ii  ("square ii")

⋙ △ abv  ("triangle abv")

Example 11.34-1 ⊙ Q denotes "circle Q."

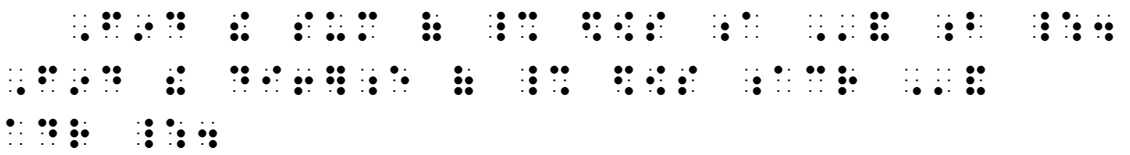
If such letters are in nonregular type, rules regarding typeform are followed and an English letter indicator is required if the variant typeform is retained.

Example 11.34-2 Compare ⬭ Z to ⬭ **Z**.



If the sign of shape has a plural or a possessive ending, an English letter indicator may be required.

Example 11.34-3 Find the sum of ∠s *a* and *b*. Find the difference of ∠s *acr* and *adr*.



Single letters "a" and "b" require an ELI when following Nemeth Code rules.(Review the definition of "single letter" in Lesson 4, section 4.10.) Letter combination "acr" requires an ELI because it is the same as a shortform; letter combination "adr" does not.

11.34.1 The Letter "m": Notation regarding angle measurement often uses the letter "m" for "measure". The letter "m" is brailled unspaced from the following symbol regardless of spacing used in the print copy.

⋙ m ∠θ  ("the measure of angle theta")

⋙ m° ∠α  ("the measure in degrees of angle alpha")

11.35 Use of the Numeric Indicator in an Enclosed List: The "enclosed list" was introduced in **Lesson 5** where it states that a numeric indicator is not used before a numeral in an enclosed list. More specifically, this rule applies to a numeral that occurs at the *beginning* of the item. A sign of shape and an identifying numeral which follows it are a single item even though a space occurs between them. In the context of an enclosed list, the numeric indicator is required for the identifying numeral because the numeral is not at the beginning of the item. Look carefully at the use and nonuse of the numeric indicator in each example below, as described in the comments.

⋙ (∠1, ∠2, ∠3) 

In this enclosed list, each numeral needs a numeric indicator because it identifies the angle symbol—the numeral does not begin the item.

⋙ (∠1, 2∠1, 3∠1) 

Only the numerals which begin an item ("2"and "3") are brailled without a numeric indicator in this enclosed list.

⋙ $(1\angle a,\ 2\angle b)$ ⠸⠣⠼⠁⠈⠐⠁⠂⠼⠃⠈⠐⠃⠜

The numerals ("1" and "2") begin each item in this enclosed list—a numeric indicator is not brailled. The letters ("a"and "b") are brailled without a letter indicator according to the rules governing identified signs of shape.

PRACTICE 11F

1. □ABCD is a square. ▱EFGH is a parallelogram. ▱ JKLM is a rhombus.
2. Compare triangles: △ADM ≅ △A'D'M'. ◿BEP ≇ ◿CFP.
3. Should △ABV be included in the set {△3, ∠ GHA, ◊2}?
4. $\angle 3 + \angle 4 = 90°$
5. $\text{m}\angle p + \text{m}\angle q = 180°$
6. $\text{m}°\angle\theta = -45$
7. $A_{\Delta DEF} = \frac{1}{2}bh$
8. $\angle ECB = \frac{1}{2}\angle ABC$

LABELED FORMATS, cont.

Displayed Material with Labels

Format for displayed mathematical expressions was introduced in **Lesson 8**. If displayed mathematical material is labeled with a unique identifier, the following details are observed.

11.36 Recognition: Displayed mathematical expressions may be labeled in print with a unique number or letter printed in a location that is visually conspicuous, often in the right margin. That label is then used in place of the actual expression later in an example or in the narrative. This allows for compact presentation of a problem.

11.36.1 Braille Layout: When a number or letter is used to label a displayed mathematical expression it is placed at the left of the expression in braille regardless of the location of the label in the print copy. The label begins in the appropriate cell for displayed material in accordance with the margin rules presented in **Lesson 8**. Nemeth Code symbols are used to label math expressions. Nonstandard typeform is disregarded. The opening Nemeth Code indicator (if needed) is placed at the end of the preceding material in order to allow the label to fall in the first cell of the displayed margin.

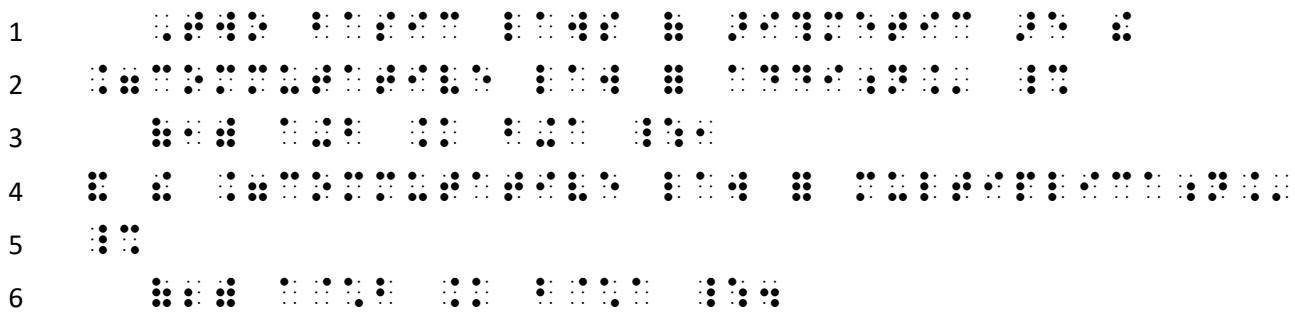
Example 11.36-1

Two basic laws of arithmetic are the
commutative law for addition

$$a + b = b + a, \qquad (1)$$

and the *commutative law for multiplication*

$$a \times b = b \times a. \qquad (2)$$



Lines 1-6: The print style is of a narrative paragraph interrupted twice by displayed math.

Lines 1-2: The paragraph begins in cell 3 and runs over in cell 1.

Line 2: The Opening Nemeth Code indicator is placed at the end of this line to assure that the item number will be the first cell to read on the next line.

Line 3: Displayed math begins in cell 3. The label is brailled first.

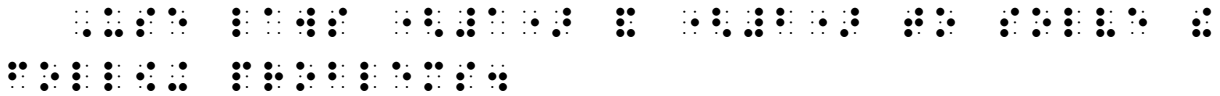
Lines 4-5: The paragraph continues in the runover cell, cell 1.

Line 6: Same comment as line 3. Note that the labels are bold in print but the typeform is disregarded in braille.

When a labeled expression is referred to within narrative, UEB symbols may be used.

Example 10.36-2

Use laws (1) and (2) to solve the following problems.



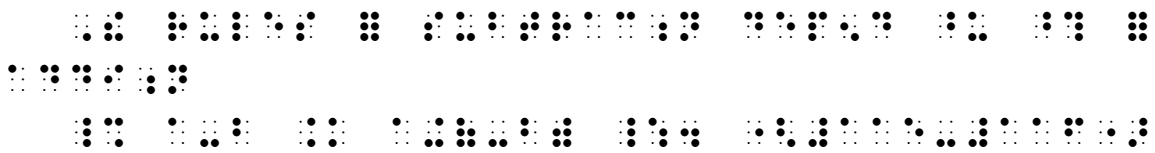
11.36.2 Transcriber's Note Required: A transcriber's note concerning the change in position of the label in the braille copy is required on the Transcriber's Notes page in every volume. *Sample:* "Identifying numbers which are printed to the right of mathematical expressions are brailled to the left." Guidelines for creating a Transcriber's Notes page are given in *Braille Formats*.

11.37 Page Number Citation: The number printed beside a displayed mathematical expression may actually be a page citation, in which case the cross reference immediately <u>follows</u> the expression, as printed. If a range of numbers is shown, you can be fairly confident that they are page numbers, but look for context clues to determine if the label is a page number citation in order to place it in its proper braille location.

Example 11.37-1

The rules for subtraction depend upon those
for addition

$$a - b = a + (-b). \qquad (115\text{-}116)$$

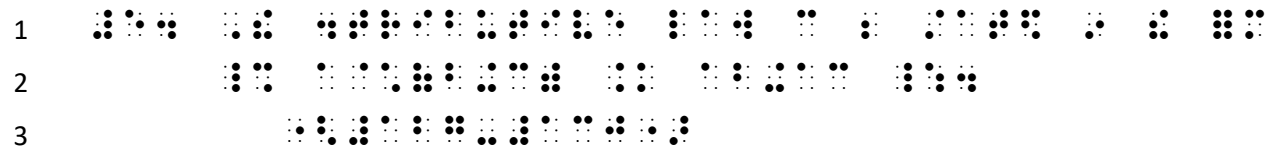


In the next example, there is no room for the citation on the line with the math. It is placed in the runover cell.

Example 11.37-2

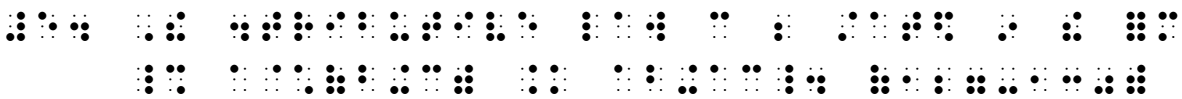
5. The distributive law can be stated in the form

$$a \times (b + c) = ab + ac. \qquad (127\text{-}130)$$



Line 1: Itemized material begins in cell 1.
Lines 2-3: Displayed margins to itemized material, (5-7)

The page numbers may be brailled in either code. If mathematical material continues after the citation, the following transcription is acceptable.



Mathematical material continues ... 

Since the location of the citation is not changed from its location in print, a transcriber's note is not needed.

Instructions: Create a Transcriber's Notes page for this practice. Include the standard statement citing usage of the Nemeth Code: "Mathematical content is transcribed according to *The Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision*, 2007-2015 Updates including the *Guidance for Transcription Using the Nemeth Code within UEB Contexts*." followed by the note regarding the change in location of identifying numbers, "Identifying numbers which are printed to the right of mathematical expressions are brailled to the left."

PRACTICE 11G

This is the quadratic equation:

$$ax^2 + bx + c = 0 \qquad (1)$$

This is the Pythagorean Theorem:

$$a^2 + b^2 = c^2 \qquad (2)$$

Which equation, (1) or (2), is used to find the length of the sides of a right triangle?

Labeled Mathematical Statements

11.38 Recognition of a Labeled Mathematical Statement: A mathematical or scientific definition, law, theorem, axiom, lemma, etc., is usually printed in a distinctive style to catch the reader's attention. It also may be set off from the main text by different margins or some other means of distinction. In this print example the definition is in boldface and is set off from the text with indented margins.

Definition **A set which can be put into one-to-one correspondence with the natural numbers is called a countable set.**

If the statement is labeled with a heading, as in the example shown above, the Nemeth Code calls it a *labeled mathematical statement* and requires the following format.

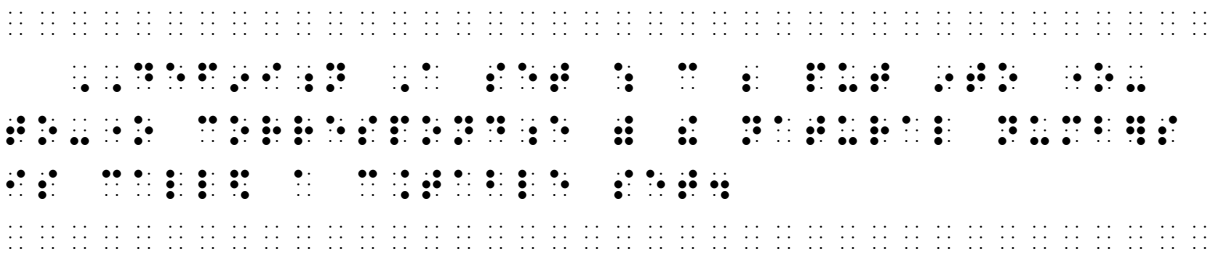
11.38.1 The Label: Transcribe the label in full capitals regardless of print style. Nonregular typeface in the label is ignored. Place the label as a paragraph heading regardless of its location in the print copy.

11.38.2 The Statement: If the text of the statement is entirely in the same typeface, the typeface is ignored in the braille transcription. This rule applies even when the statement is entirely in UEB.

11.38.3 Spacing and Margins: A line is left blank before the beginning and after the end of the entire labeled statement. Normal paragraphing (3-1) is applied, with the label beginning the paragraph.

Example 11.38-1

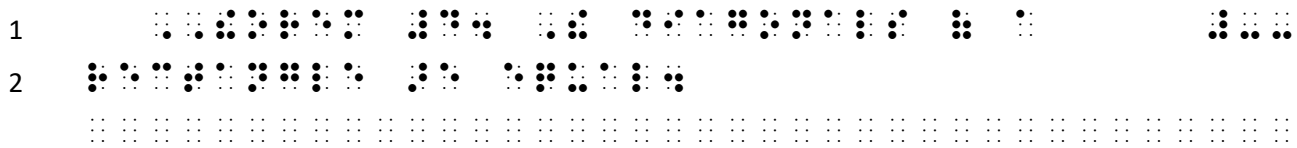
Definition **A set which can be put into one-to-one correspondence with the natural numbers is called a countable set.**



In print, the label "Definition" is a marginal heading. Only the first letter is capitalized and the word is in boldface. In braille, typeface of the label is disregarded, the label is fully capitalized and is placed as a paragraph heading beginning in cell 3. The definition is printed entirely in boldface. Uniform typeface is disregarded in braille. A blank line precedes and follows the labeled statement.

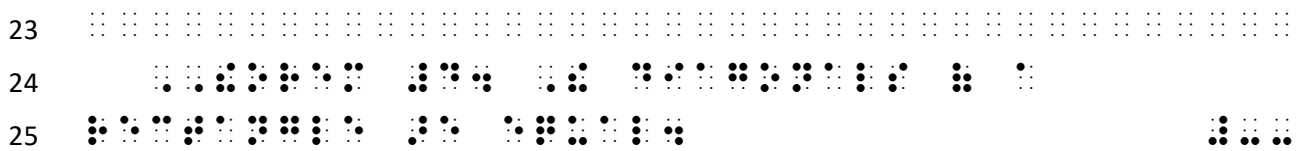
A labeled statement may begin on line 1 of the braille page.

Example 11.38-2 **Theorem 4.** *The diagonals of a rectangle are equal.*



In print, "Theorem 4" is bold and the statement is in italics.

A labeled statement may end on line 25 of the braille page.

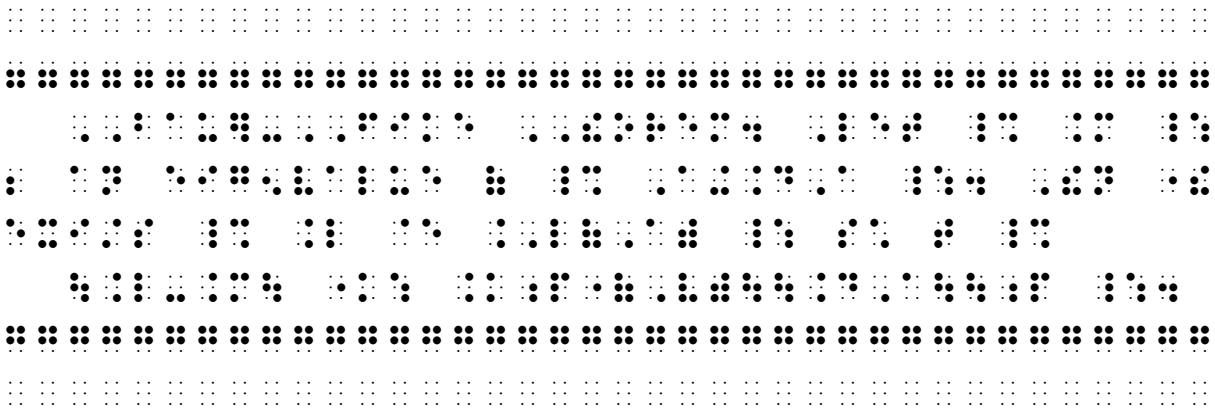


If the statement is printed in a box, follow *Braille Formats* guidelines regarding blank lines.

Example 11.38-3

Bauer–Fike Theorem. Let μ be an eigenvalue of $A + \delta A$. Then there exists $\lambda \in \Lambda(A)$ such that

$$|\lambda - \mu| \leq \kappa_p(V)\|\delta A\|_p \, .$$



The labeled statement is boxed. Box lines are retained in braille for distinction. The label "Bauer-Fike Theorem." is printed as a paragraph heading. Only the initial letters are capitalized and the label is in boldface. In braille, typeface is disregarded and the label is fully capitalized. The definition itself is printed in normal typeface, with the exception of the letters in the mathematical expressions which are in italics. In braille, italics applied to a variable are disregarded.

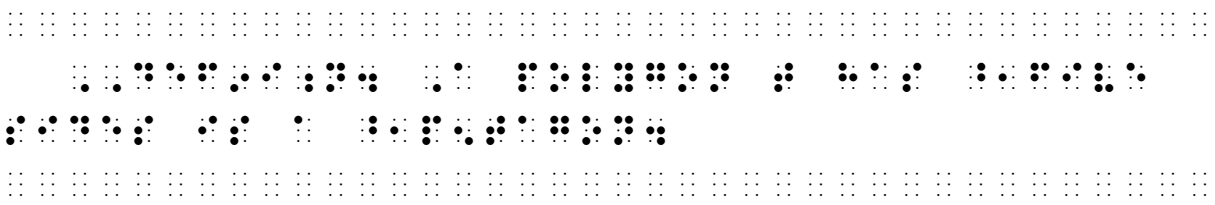
Review unfamiliar symbols. This example above includes Greek letters mu, delta, lambda (both lowercase and uppercase), and kappa as well as the comparison sign for

"membership". You can assume the letter A is the English letter, not a Greek Alpha. Several uppercase Greek letters are generally not used as math symbols because they look identical to certain uppercase Latin (English) letters. Review the Greek Alphabet Table in ***Lesson 5****.*

11.39 Significant Typeface: If, in the body of the labeled statement, a word or phrase is singled out for special attention by using a non-regular typeface (for the purpose of definition or other elaboration), the change in typeface is retained in braille. Use the appropriate UEB or Nemeth Code typeface indicators. UEB indicators are used outside of the code switches; Nemeth Code indicators are used inside the code switches.

Analysis: In the next example, the label "Definition." is printed in boldface. Only the first letter is capitalized. In braille, typeface is disregarded and, instead, the label is fully capitalized. The statement is printed entirely in italics but two words are emphasized in bold italics. In braille, the superfluous typeface (italics) is disregarded but the typeface of the emphasized words (boldface) is retained.

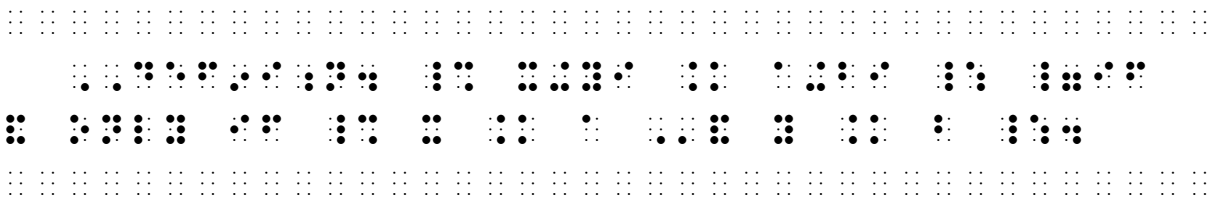
Example 11.39-1 **Definition.** *A polygon that has* ***five*** *sides is a* ***pentagon.***



Recall from **Lesson 7** that a switch from UEB to Nemeth Code terminates the effect of a UEB typeform indicator.

Analysis: In the next example, the label "Definition" is printed in italics and only the first letter is capitalized. In braille, the label is fully capitalized. The statement is printed entirely in boldface. This superfluous typeface is ignored in braille. One phrase is emphasized by underlining. The underlining is retained in braille.

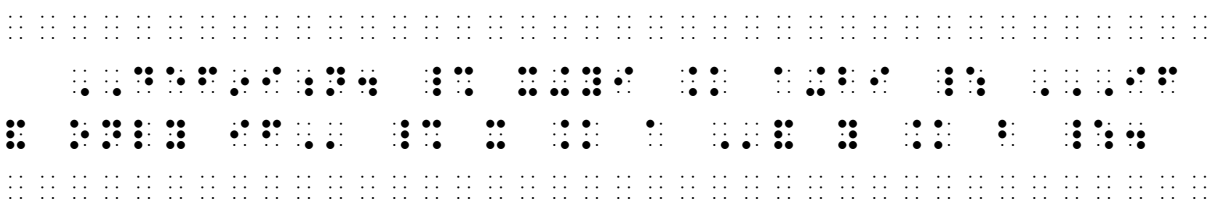
Example 11.39-2 *Definition* $\boldsymbol{x+yi=a+bi}$ <u>if and only if</u> $\boldsymbol{x=a}$ **and** $\boldsymbol{y=b}$**.**



The underlining of the passage is implicitly terminated by the switch to Nemeth Code.

Implicit termination does not apply to capitalization. A fully capitalized passage must be explicitly terminated.

Example 11.39-3 *Definition* $\boldsymbol{x+yi=a+bi}$ **IF AND ONLY IF** $\boldsymbol{x=a}$ **and** $\boldsymbol{y=b}$**.**



PRACTICE 11H

Labeled Mathematical Statements

Pythagorean Theorem *In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.*

$$c^2 = a^2 + b^2$$

DEFINITION A positive number expressed in the form: $a \times 10^n$, where $1 \leq a < 10$ and n is an integer is said to be written in **scientific notation.**

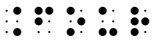
Typeform Indicators for Mathematical Words and Phrases

11.40 Italic and Boldface Typeform Indicators: When significant typeform is retained inside of the Nemeth Code switches, Nemeth Code typeform indicators are used. **Lesson 7** introduced the Nemeth Code typeform indicators for letters, numerals, symbols, and compound expressions. When typeform is retained for a mathematical *word or phrase*, the following rules apply.

11.40.1 One Word in Italics or Boldface

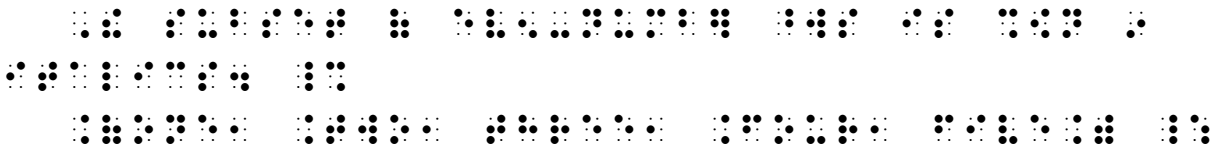
For One Word	
Italic Typeform Indicator	
Opening Boldface Type Indicator	
Closing Boldface Type Indicator	

The one-cell italic typeform indicator is unspaced from the following word.

⋙ *four* 

Example 11.40-1 The subset of even-number words is shown in italics.

{one, *two*, three, *four*, five}

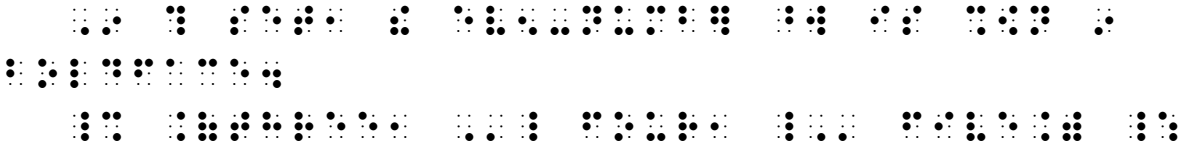


The three-cell boldface type indicators are preceded and followed by a space and must not stand alone on a line.

⋙ **four** 

Example 11.40-2 In this set, the even-number word is shown in boldface.

{three, **four,** five}

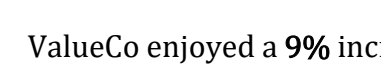


Note: The one-cell boldface typeform indicator of the Nemeth Code  *is applied to a word only in the context of a compound expression. See* ***Lesson 7****.*

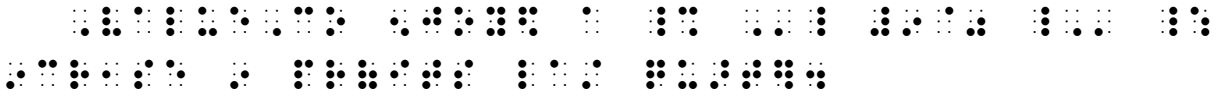
When typeform is retained for a fraction, or for a numeral that is associated with a mathematical symbol such as a percent sign or monetary symbol, the three-cell typeform indicator is used.

⋙ $\mathbf{\frac{9}{100}}$

⋙ **9%**



Example 11.40-3 ValueCo enjoyed a **9%** increase in profits last quarter.

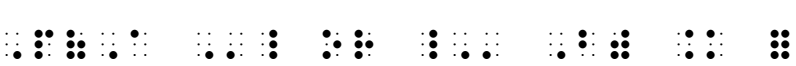


The BANA Nemeth Code Technical Committee is discussing details regarding code switching requirements for freestanding, unmodified mathematical numbers in narrative context when such numbers are printed in a significant typeface. At this time, no such examples are shown.

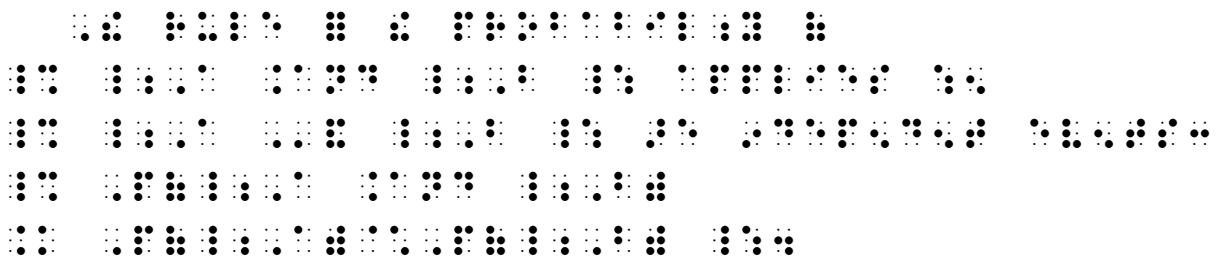
Significant typeform with a single word is often found in the study of probability. For example,

⋙ $P(A \textit{ and } B) = 0$

⋙ $P(A \textbf{ or } B) = ?$



Example 7.40-4 The rule for the probability of **A** *and* **B** applies when **A** and **B** are independent events: $P(\mathbf{A} \textit{ and } \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B})$.



A switch to Nemeth Code is required for the boldface letters because they have mathematical significance. Italic typeform is disregarded for the single letter "P". The italicized word "and" is part of the mathematical expression and is brailled in Nemeth Code. The word "and" printed in normal type is part of the sentence structure and is brailled in UEB. The linked expression is divided before the equals sign because it will not fit entirely on one line (this is a 38-cell line).

11.40.2 A Phrase Italics or Boldface

For a Phrase	
Opening Italic Type Indicator	
Closing Italic Type Indicator	
Opening Boldface Type Indicator	
Closing Boldface Type Indicator	

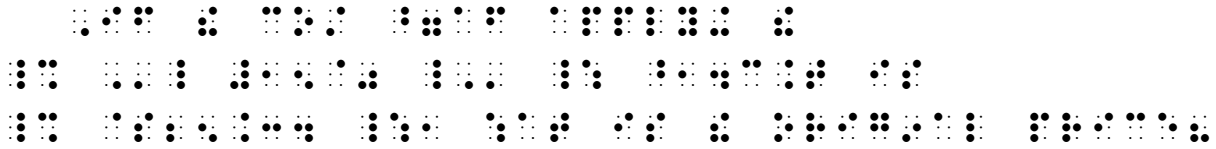
The three-cell typeform indicators are preceded and followed by a space and must not stand alone on a line. When both indicators are required for the same word or phrase, they are unspaced from each other and are closed in the opposite order as opened.

⋙ *4.9 sq. **ft.*** 

Assume the italics is significant in this illustration.

11.41 Code Switching Within an Emphasized Passage: If code switching is necessary within an emphasized technical passage, the beginning typeform indicators are repeated after each switch to show that emphasis continues. Switching from Nemeth Code to UEB does not implicitly terminate a Nemeth Code typeform – the appropriate closing typeform indicator must be brailled before terminating Nemeth Code.

Example 11.41-1 If the cost **after applying the 15% discount** is $25.34, what is the original price?



It is first determined that the boldface should be retained because its purpose here is to focus attention on the phrase.

Line 1: UEB boldface passage indicator applies to three words. The switch from UEB to Nemeth Code terminates the effect of the UEB typeform indicator without the need for a UEB termination indicator.

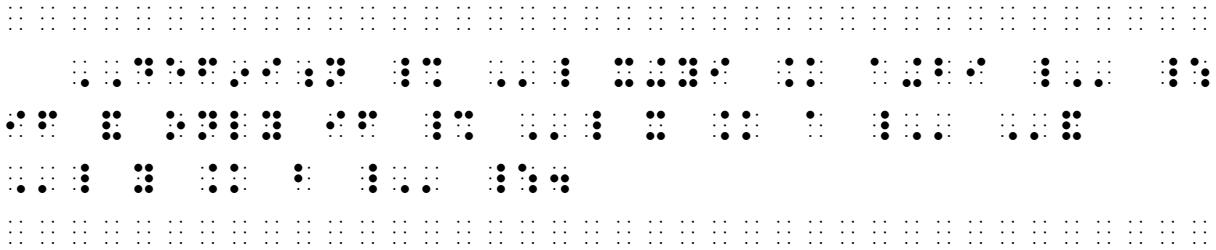
Line 2: The boldface indicator of the Nemeth Code is used for the boldfaced mathematical item, 15%. The Nemeth Code closing boldface indicator is required before switching out of Nemeth Code.

Line 2: The nonregular typeform continues after the termination of the mathematical portion, so a UEB typeform indicator must be re-entered.

11.42 Revisiting Typeform in Labeled Statements: Further examples illustrate the use of UEB and Nemeth Code typeform indicators in the context of labeled statements.

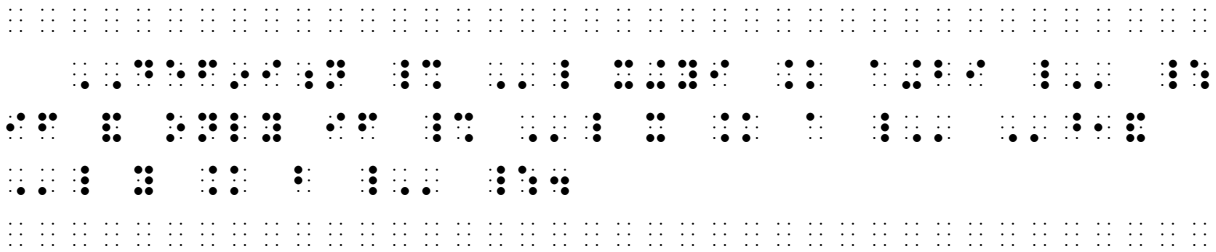
Analysis: In the next example, the label and the statement are printed in a consistent typeface (italics). Each of the three equations is printed in boldface italics. The italic typeform is ignored because italics is universally applied to this statement, but the boldface is retained because it is in a variant typeform in relation to the rest of the statement. The boldface type indicators of the Nemeth Code are used for each equation.

Example 11.42-1 *Definition* $\boldsymbol{x + yi = a + bi}$ *if and only if* $\boldsymbol{x = a}$ *and* $\boldsymbol{y = b}$.



Analysis: In the next similar example, the variant typeform continues in the UEB portion (the word "and" also printed in boldface italics). After terminating boldface in the Nemeth portion of this phrase, the single-word switch indicator initiates UEB and the UEB boldface word indicator is brailled. Boldface continues with an opening Nemeth Code typeform indicator, finally closing before the Nemeth Code terminator.

Example 11.42-2 *Definition* $\boldsymbol{x + yi = a + bi}$ *if and only if* $\boldsymbol{x = a}$ ***and*** $\boldsymbol{y = b}$.



PRACTICE 11I

1) In solution b, *only the* $x^2 > 0$ *values* apply.

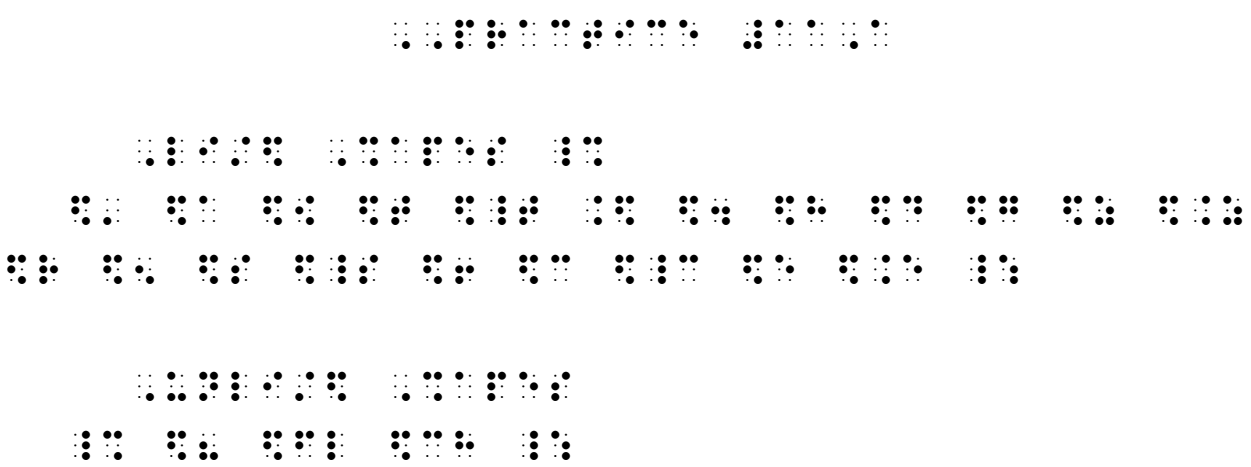
2) The *reciprocal* of a fraction is the fraction turned upside-down. For example, **the reciprocal of** $\frac{2}{3}$ **is** $\frac{3}{2}$.

3) The *Specific Addition Rule of Probability* can be expressed as follows:

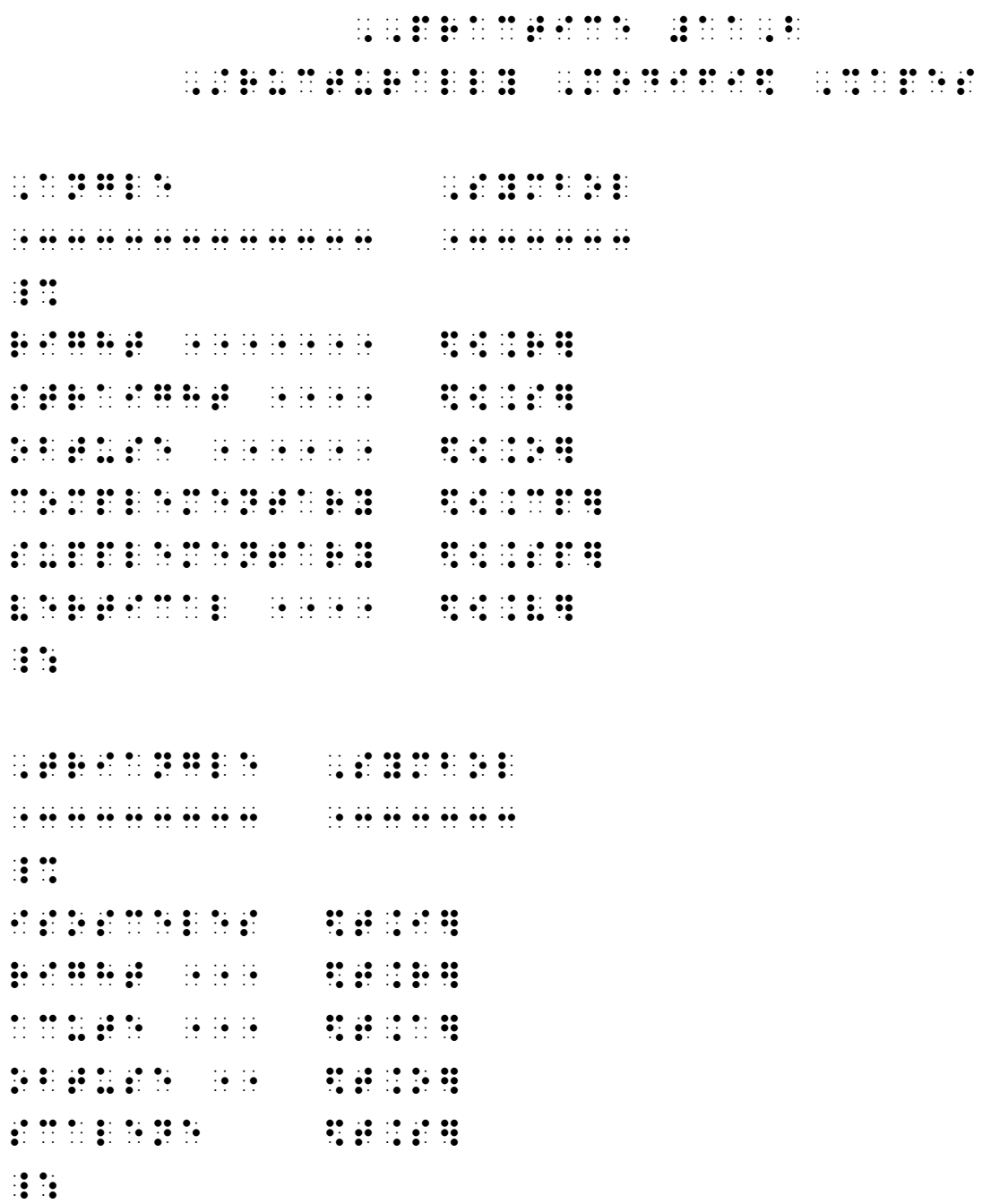
$$P(A \text{ or } B) = P(A) + P(B)$$

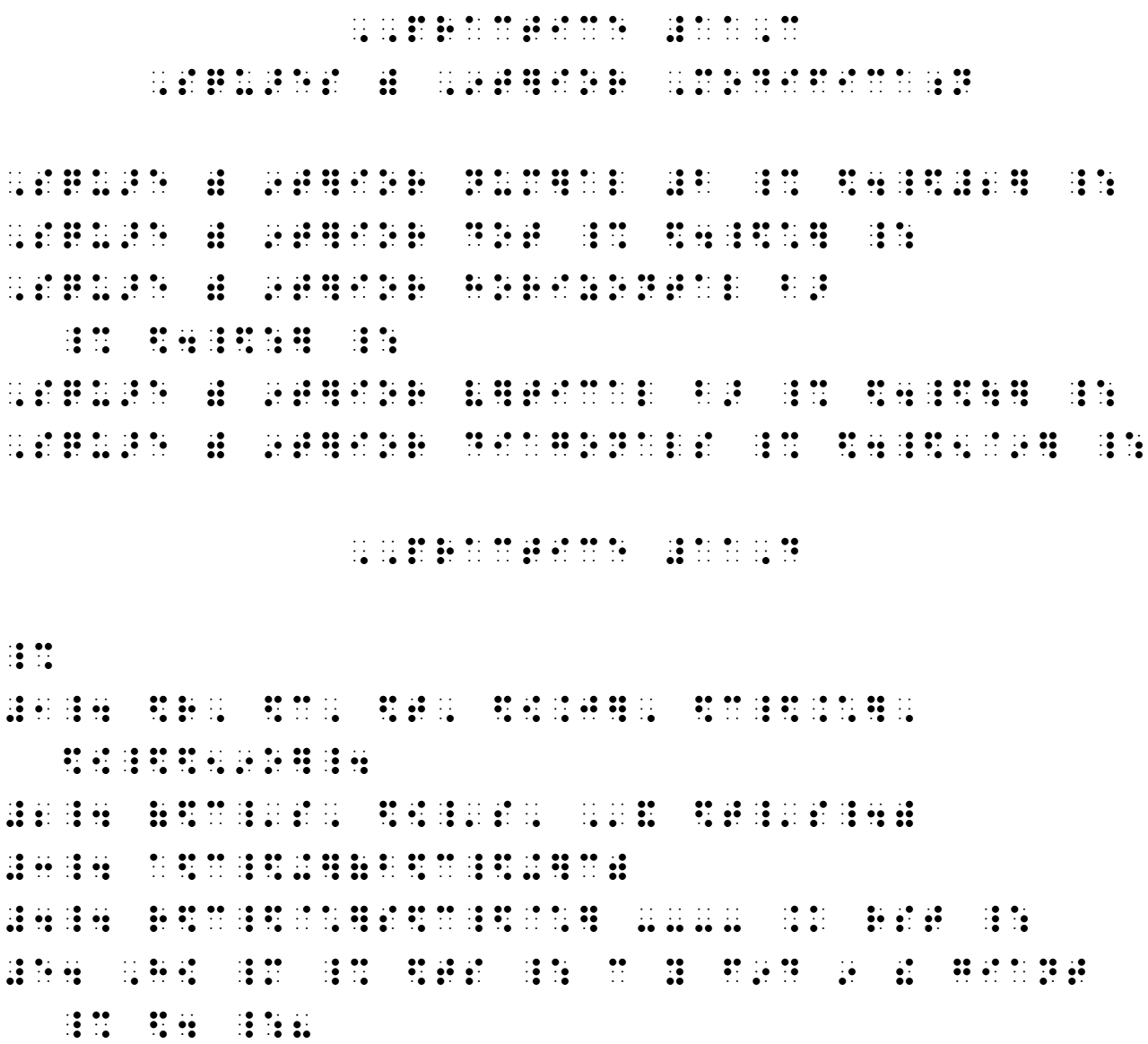
For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL

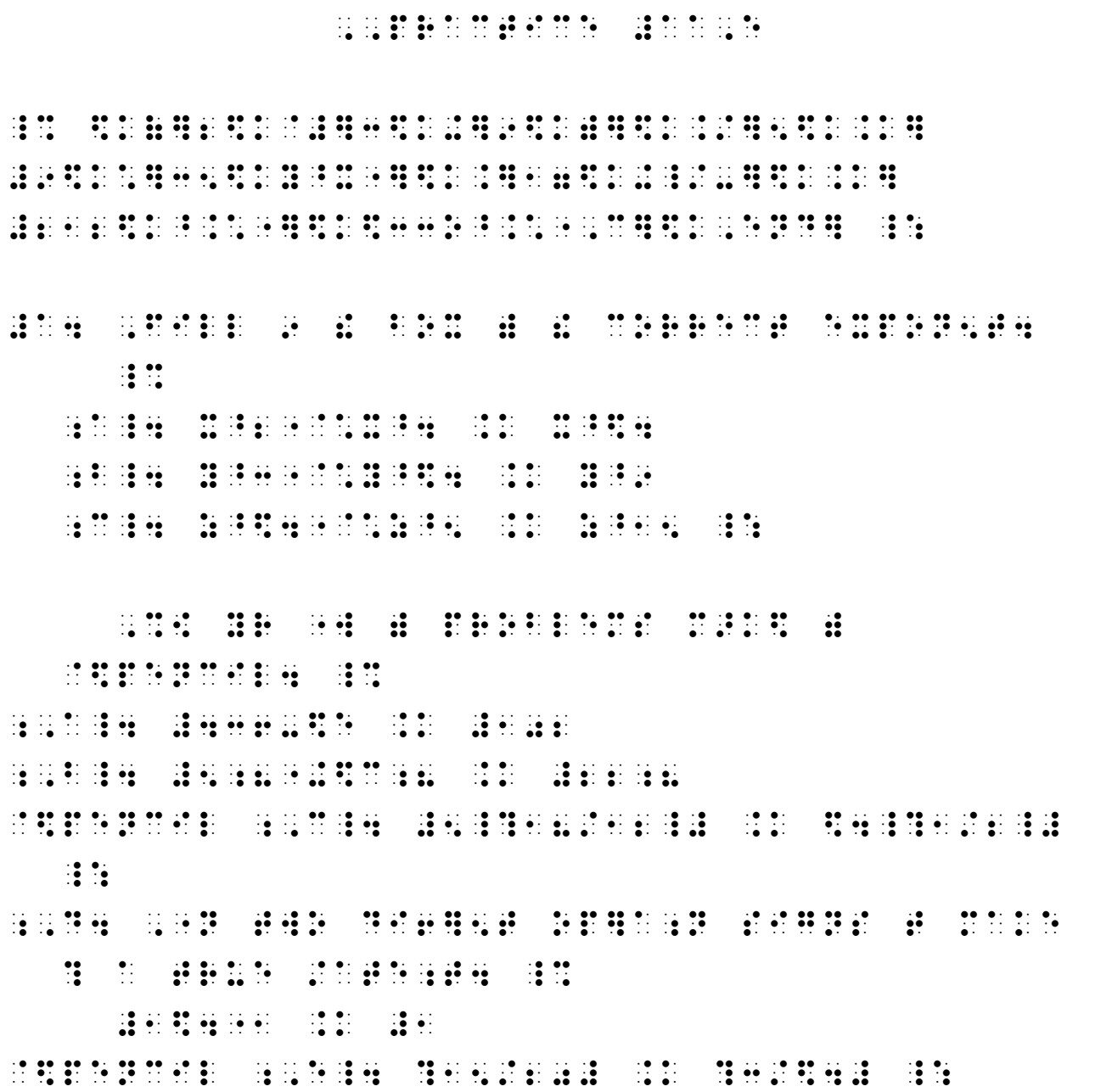


The first opening Nemeth Code indicator may also be placed at the beginning of the series of shapes on line 4.

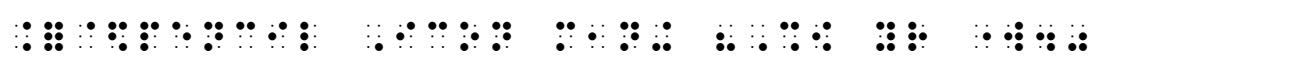


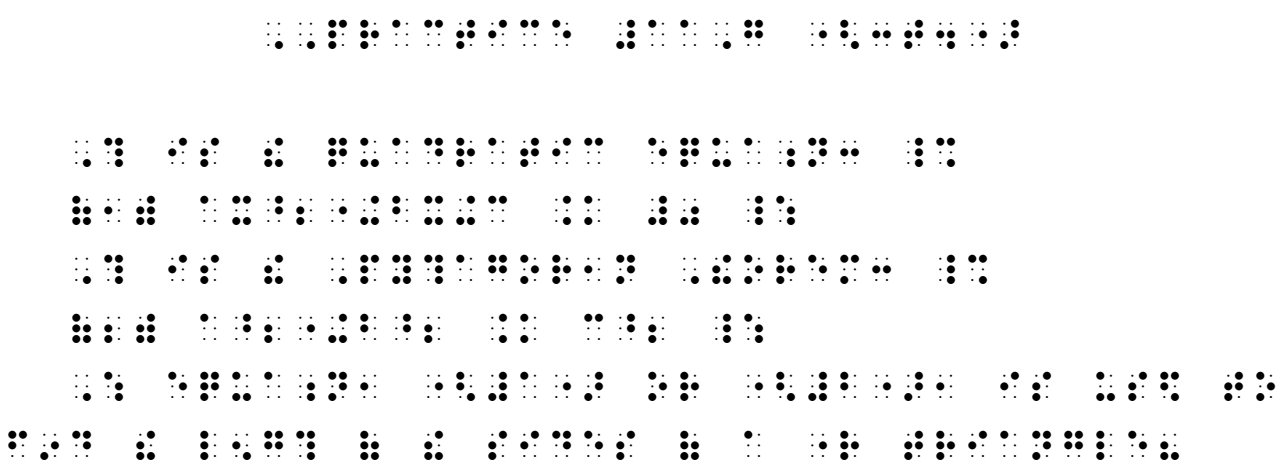


The opening Nemeth Code indicator does not replace the required blank line following the centered heading.

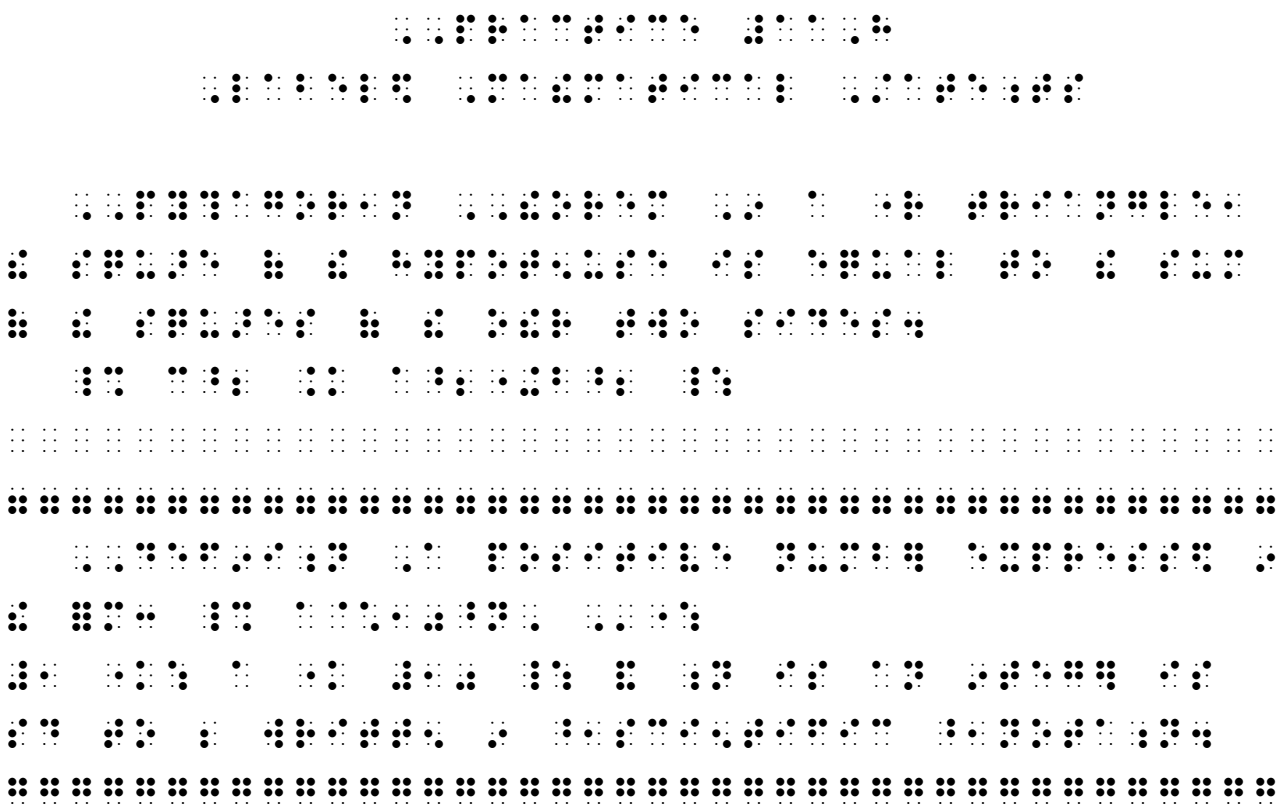


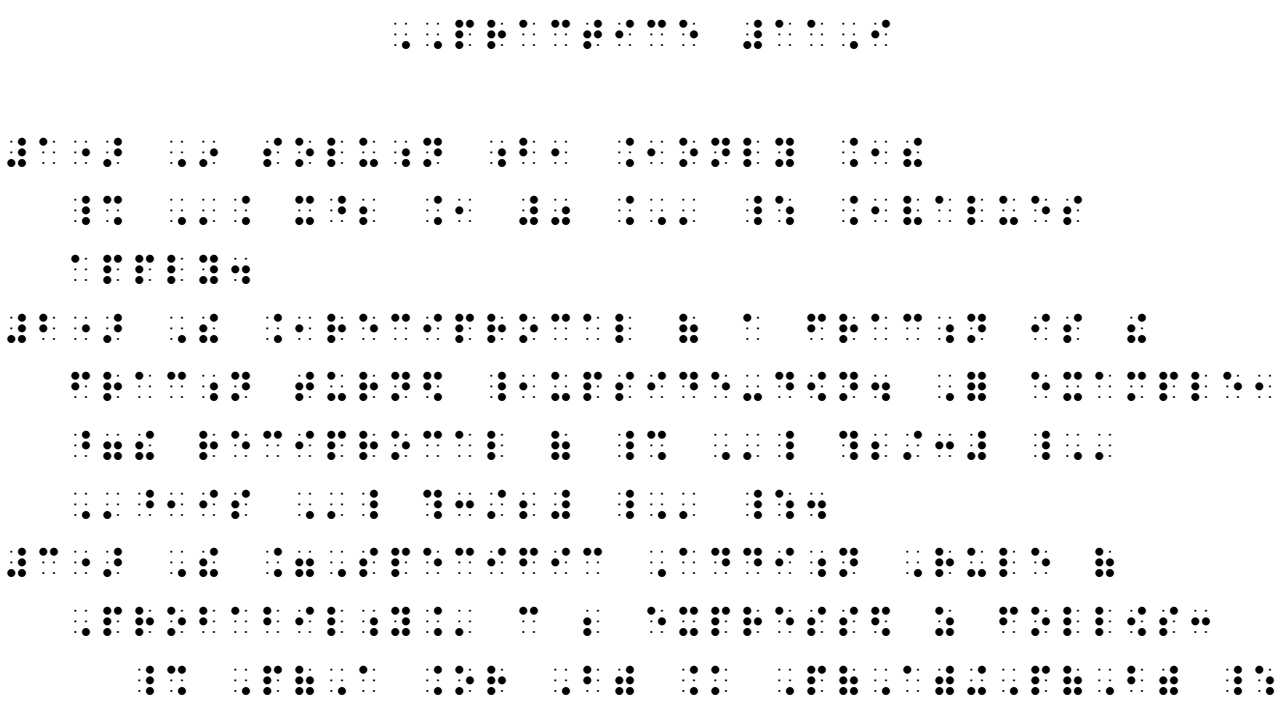
Special Symbols page:





This is three paragraphs. Math displayed to a 3-1 narrative paragraph begins in cell 3.





EXERCISE 11

Exercise 11 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 12.