LESSON 8

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

LEVEL INDICATORS

8.1 Definition: A mathematical expression can contain symbols placed above or below the baseline level (the normal line of type in print). Superscripts appear above the baseline; subscripts appear below the baseline. Here are some expressions which contain superscripts and subscripts. The baseline is marked for orientation.

__baseline__ 10^5 H_2O $^-4$ °C πr^2 $^{235}_{92}U$ __baseline__

Note: The font size is increased for the isolated examples in this lesson to help in determining the levels.

Superscripts

8.2 Superscript Level Indicator: In braille, indicators are used to identify the level of a superscript or subscript. The superscript level indicator is used to show that the symbols immediately following it appear on the first level above the baseline of writing.

Superscript Indicator	⠘

⫸ 10^7

⫸ $y = [\, g(x)]^n$

The characters in the superscript are spaced according to the rules of the Nemeth Code.

⫸ $3^{-0.05T}$

Remember, a numeral does not need a numeric indicator when it follows and is unspaced from a mathematical character or indicator. A letter, a shortform letter combination, or a Roman numeral does not need an English letter indicator unless it is preceded by a space or by one or more punctuation marks and followed by a space or by one or more punctuation marks.

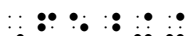
⫸ $x\ x^3$

Example 8.2-1 Statements i^2 and ii^2.

Example 8.2-2 Theorems I^2 and V^2.

8.2.1 Abbreviations with Superscripts: When a superscript applies to an abbreviation, the level indicator is unspaced from the abbreviation.

⋙ $19\ \text{in.}^3$ 

⋙ Fe^{++} 

Reminder about single-letter abbreviations: Rules for the nonuse of the ELI with mathematical "single letters" do not apply to abbreviations. A single-letter abbreviation from the English alphabet that does not have a related period must always begin with an English letter indicator.

⋙ $5.7\ \text{m}^2$ 

The English letter indicator is used for a single-letter abbreviation without a period, even when unspaced from an indicator.

PRACTICE 8A

$6x^2$	$24^?$	x^{2a}
π^2	$(y-k)^3$	8^{10}
n^m	$A^{(k+1)+m}$	$q^{b(w-1)}$
$k^{0.27}$	$e^{3{,}000}$	e^-

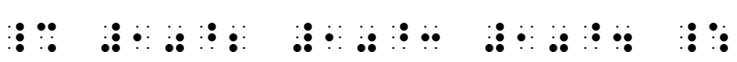
Measurements of **length** are measurements in **one dimension**. They are labeled as cm, ft, m, etc. Measurements of **area** are measurements in **two dimensions**. They are labeled as cm^2, ft^2, m^2, etc. Measurements of **volume** are measurements in **three dimensions**. They are labeled as cm^3, ft^3, m^3, etc.

The 3-D coordinate system is often denoted by $\mathbb{R}^3$. Likewise, the 2-D coordinate system is often denoted by $\mathbb{R}^2$. A general *n*-dimensional coordinate system is often denoted by $\mathbb{R}^n$.

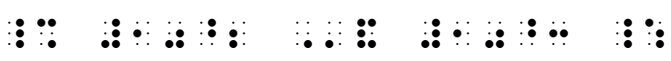
8.3 Returning to the Baseline Level: When a superscript is the last item in the expression, the following circumstances bring the reader back to the baseline level: a space, a comma followed by a space, or a punctuation indicator. "A space" includes the space before a Nemeth Code termination indicator and/or transition to a new braille line.

— A space returns the reader to the baseline.

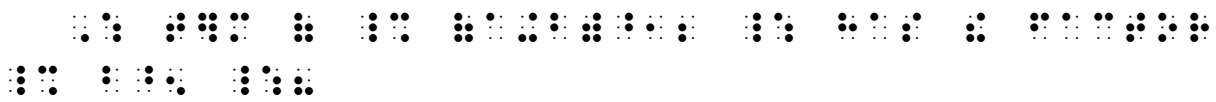
Example 8.3-1 $10^2\ 10^3\ 10^4$



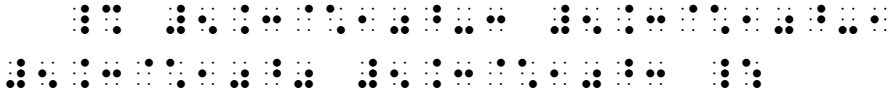
Example 8.3-2 10^2 and 10^3



Example 8.3-3 Which term of $(a+b)^{12}$ has the factor b^5 ?

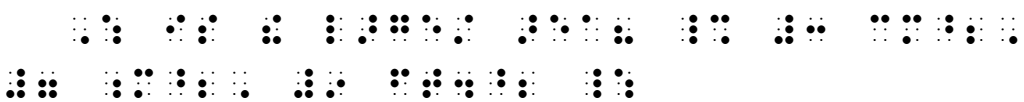


Example 8.3-4 $5.3\times10^{-3}\ \ 5.3\times10^{-1}\ \ 5.3\times10^{0}\ \ 5.3\times10^{3}$

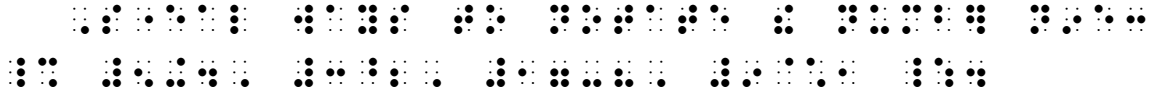


— A comma followed by a space returns the reader to the baseline.

Example 8.3-5 Which is the largest area? $3\ \text{cm}^2$, $7\ \text{m}^2$, $9\ \text{ft.}^2$

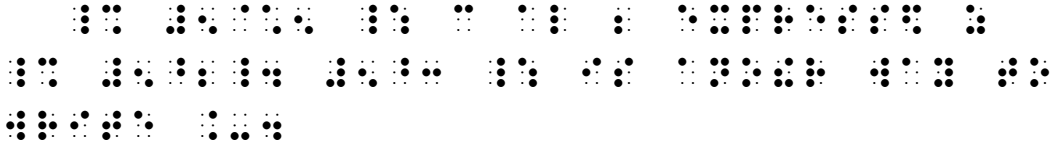


Example 8.3-6 Several ways to notate the number nine: $5+4,\ 3^2,\ 17-8,\ 9\times1.$



— A punctuation indicator returns the reader to the baseline.

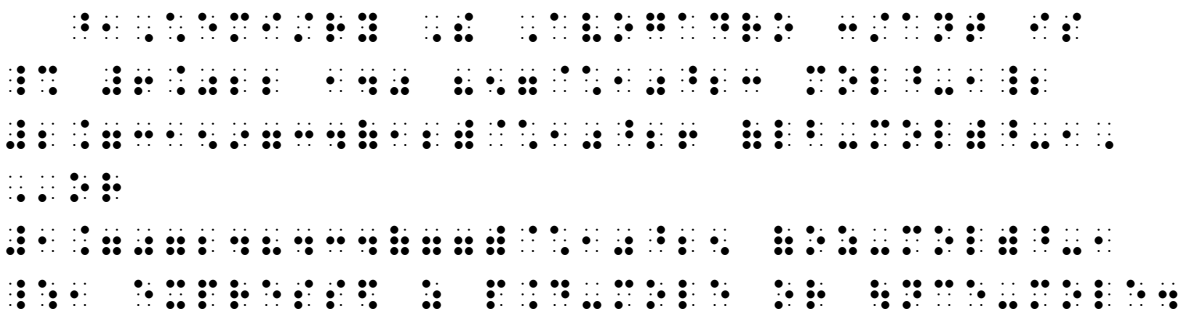
Example 8.3-7 5×5 can also be expressed as 5^2. 5^3 is another way to write ___.



Notice how the baseline level is re-established after each of the six superscripts in the next example.

Example 8.3-8

Chemistry The Avogadro constant is $6.022\ 140\ 857 \times 10^{23}\text{mol}^{-1}$; $2.73159734(12) \times 10^{26}(\text{lb-mol})^{-1}$, or $1.707248434(77) \times 10^{25}(\text{oz-mol})^{-1}$, expressed as pound-mole or ounce-mole.

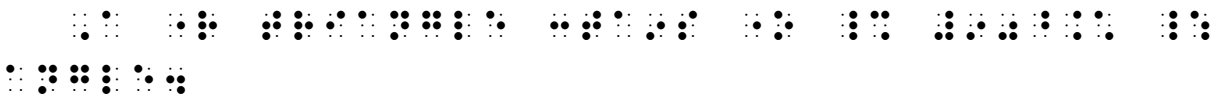


The baseline is re-established as follows: the space before each abbreviation ("mol", "(lb-mol)", and "(oz-mol)"); the punctuation indicator before the semicolon; the math comma and the following space (or, in this case, transition to a new line); the space before the Nemeth Code terminator.

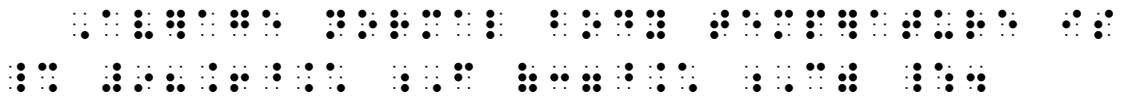
8.4 Raised Hollow Dot: The hollow dot in the superscript position may be used to represent degrees of angle or degrees of temperature. Transcription of the "degrees" symbol requires a switch to Nemeth Code.

Raised Hollow Dot representing Degrees	°	

Example 8.4-1 A right triangle contains one 90° angle.



Example 8.4-2 Average normal body temperature is 98.6° F (37° C).



"F" and "C" are spaced according to Nemeth Code rules for abbreviations.

PRACTICE 8B

1. Tell what number each of the following names: 6^2, 5^{-3}, and $(3.15)^4$.
2. Which expression denotes the area of a circle?
   a. 360°
   b. πr^2
   c. πd
3. The symbol for "feet squared" is ft^2. $1\ \mathrm{ft} \times 1\ \mathrm{ft} = 1\ \mathrm{ft}^2$
4. 6.02×10^{23} is an approximation of "Avogadro's number".

Introduction to the Baseline Indicator

8.5 Function of the Baseline Indicator: In an unspaced expression, a return to the baseline level is brought about by the use of the baseline indicator.

Baseline Indicator	⠐

⫸ $a^2 + b^2 + c^2$ ⠁⠘⠆⠐⠬⠃⠘⠆⠐⠬⠉⠘⠆

Example 8.5-1 Draw a 30°-60°-90° triangle.

⠠⠙⠗⠁⠺ ⠁ ⠸⠩ ⠼⠒⠴⠘⠨⠡⠐⠤⠖⠴⠘⠨⠡⠐⠤⠔⠴⠘⠨⠡ ⠸⠱
⠞⠗⠊⠁⠝⠛⠇⠑⠲

Notice that the baseline indicator is the same symbol as the multipurpose indicator—dot 5. The indicator's function is determined in context.

8.5.1 Abbreviations with Superscripts: When returning to the baseline after an abbreviation with a superscript, no space is left before the baseline indicator.

Example 8.5-2 Adding cubic meters: $2\ \mathrm{m}^3 + 2\ \mathrm{m}^3 = 4\ \mathrm{m}^3$

⠠⠁⠙⠙⠬ ⠉⠥⠃⠊⠉ ⠍⠑⠞⠻⠎⠒
⠸⠩ ⠼⠆ ⠰⠍⠘⠒⠐⠬⠆ ⠰⠍⠘⠒ ⠨⠅ ⠼⠲ ⠰⠍⠘⠒ ⠸⠱

Reminder: The superscript indicator is unspaced from the abbreviation.

Compare the spacing in the next two examples.

⋙ 144 ft + 144 ft

A space precedes and follows the abbreviation "ft". The operation sign is unspaced from the number which follows.

⋙ 144 ft^2 + 144 ft^2

A space precedes the abbreviation "ft". The superscript is unspaced from the abbreviation. There is no space before the baseline indicator.

PRACTICE 8C

(1) $4(x-y)^3 - 2(x-y)^3$

(2) $3a^3b + 6a^6b^2 + 9a^9b^3$

(3) 98.6° F

(4) $V = 2\pi^2 Rr^2$

(5) x^n-dimensional system

(6) $6^2 \times 6^3 = 6^{2+3} = 6^5$

(7) 144 ft^2 + 144 ft^2 = ...

(8) Given that $(x+2)$ and $(x-3)$ are factors of $f(x)$, show that $a = 3$ and $b = -10$.

8.6 Higher Levels of Writing: Superscripts may carry superscripts of their own. In such cases, the superscript indicator is doubled, tripled, etc. to indicate superscripts on the second, third, or higher levels of writing.

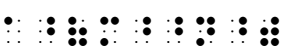
Superscript with Superscript		*(two levels above the baseline)*
Superscript with Superscript with Superscript		*(three levels above the baseline)*

⋙ n^{a^b} 

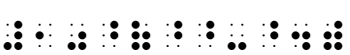
⋙ $n^{a+1^{b+1^{c+1}}}$ 

If the print copy is difficult to analyze, a straightedge and a magnifier will help to determine the levels. In general, the print font gets smaller as the level gets higher.

8.6.1 Combinations: Each level indicator is read *as it relates to the baseline level*. The effect of one level indicator is terminated by another level indicator. Keep this in mind as you read the following examples.

⋙ $a^{(m^n)}$ 

The opening and closing parentheses and the letter "m" are one level above the baseline; the letter "n" is two levels above the baseline.

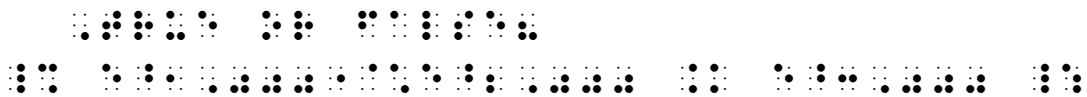
⋙ $10^{(-4)}$ 

The opening and closing parentheses and the numeral "4" are one level above the baseline; the minus sign is two levels above the baseline.

Example 8.6-1 Will $E[x^2] = e^{2(a+b^2)}$?

The square brackets are on the baseline level. The second superscript is $2(a + b^2)$ *which means that the final "2" is at the super-superscript level.*

Example 8.6-2 True or false? $e^{1,000} \times e^{2,000} = e^{3,000}$



When a space brings the reader back to the baseline, a baseline indicator is not needed.

8.7 Certain Raised Signs: Some signs are printed in a raised position but are not considered to be superscripts.

8.7.1 Raised Ordinal: *Braille Formats* states that raised ordinal endings are not considered to be superscript. This guideline is also followed in a technical transcription, whether following the rules of UEB or of Nemeth Code.

Example 8.7-1 Name the 6th and 7th item in the series.

Example 8.7-2 What is the 3nth degree?

8.7.2 Prime Sign: The prime sign, introduced in **Lesson 1**, appears to be raised in print but it assumes the same level as the number or letter to which it applies.

Prime Sign	'	⠄

⋙ $x'^{\,2}$ ⠭⠄⠘⠆ *The prime sign belongs with the letter x. Only the number 2 is in the superscript position.*

⋙ $A^{2''}$ ⠠⠁⠘⠆⠄⠄ *The prime signs belong with the number 2 in the superscript.*

Example 8.7-3 plane angle $\alpha = 30°2'8''$

⠏⠇⠁⠝⠑ ⠁⠝⠛⠇⠑ ⠸⠩ ⠨⠁ ⠨⠅ ⠼⠒⠴⠘⠨⠡⠐⠆⠄⠦⠄⠄ ⠸⠱

The prime signs are not superscripts. They apply to numbers at the baseline of writing.

8.7.3 Apostrophe-s: In an apostrophe-s ending, the apostrophe is at the same level as the "s". Because a punctuation indicator returns the reader to the baseline, a level indicator is inserted to maintain the level of the following "s" in the next example.

⋙ $\mathrm{A}^{m+m+m\text{'}s}$ ⠠⠁⠘⠍⠬⠍⠬⠍⠘⠸⠄⠎

Compare to this example where the "s" and its apostrophe are printed on the baseline level.

⋙ A^{m+m+m}'s ⠠⠁⠘⠍⠬⠍⠬⠍⠸⠄⠎

PRACTICE 8D

1. Use a calculator to find 9^{9^9}.
2. Find the r$^{\text{th}}$ term of $(x+y)^n$.
3. Label the x^2 's and x^3 's.
4. What is the meaning of x''^3?
5. Simplify: $(x^3 - y^3)^2 - (x^3 + y^3)^2$.
6. $x^{y^n z}$ or $x^{y^2 z}$

Subscripts

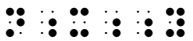
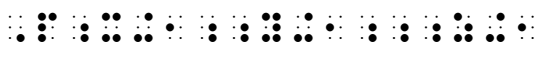
8.8 Subscript Level Indicators: *Except* as stated in **8.10** below, the subscript level indicator is used to show that the symbols immediately following it appear on the first level below the baseline of writing.

Subscript Indicator	⠰

- ⋙ f_n 
- ⋙ $a_{(k+1)}$ 

Subscripts may carry subscripts of their own. In such cases, the subscript level indicator is doubled, tripled, etc. to indicate subscripts on the second, third, or lower levels.

Subscript with Subscript	⠰⠰	*(two levels below the baseline)*
Subscript with Subscript with Subscript	⠰⠰⠰	*(three levels below the baseline)*

- ⋙ n_{x_y} 
- ⋙ $P_{x+1_{y+1_{z+1}}}$ 

Note that the subscript indicator ⠰ is the same symbol as the English letter indicator. The indicator's function is determined in context.

Recall that an English letter indicator is not used in an unspaced mathematical expression unless the letter is printed in a mathematically-significant typeform. Compare these transcriptions of the letter "i" in regular type and bold type:

- ⋙ $3i$ 

  Spoken: three i

- ⋙ 3_i 

  Spoken: three sub i

- ⋙ $3\boldsymbol{i}$ 

  Spoken: three bold i

- ⋙ $3_{\boldsymbol{i}}$ 

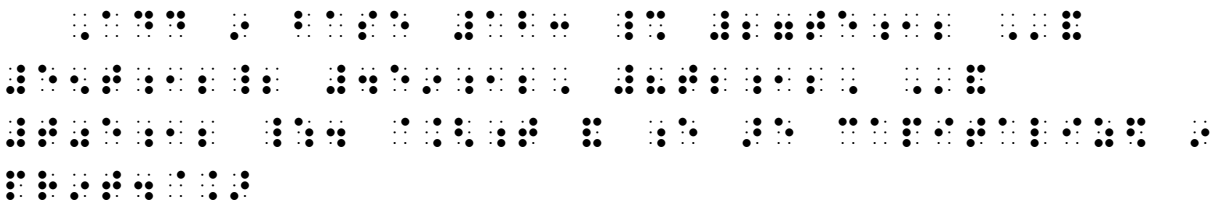
  Spoken: three sub bold i

PRACTICE 8E

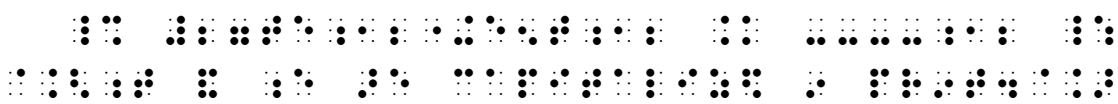
3_c	10_8	$?_3$	x_{2+k}
δ_{ij}	$10_?$	$\mathrm{Ca(OH)_2}$	y_{-2}
y_{n_k}	3_{five}	P_{3n}	a_{m1}

8.9 Returning to the Baseline Level: The same circumstances discussed with superscripts bring the reader back to the baseline level from a subscript: a space, a comma followed by a space, a punctuation indicator, or a baseline indicator. "A space" includes the space before a Nemeth Code termination indicator and/or transition to a new braille line.

Example 8.9-1 Add in base 12: $27\mathrm{TE}_{12}$ and $\mathrm{E5T}_{12}$; $4\mathrm{E9}_{12}$, $8\mathrm{T2}_{12}$, and $\mathrm{T0E}_{12}$.



Example 8.9-2 $27\mathrm{TE}_{12} + \mathrm{E5T}_{12} = \_\_\_{12}$



Note that the level indicator is unspaced from the long dash to which it applies.

Example 8.9-3 $(R_H T_H) + (R_S T_S)$

⠰ *is the subscript indicator throughout this expression. Letters H and S are subscripts.*

PRACTICE 8F

1. Name the numeral in base ten equal to:
   a. 47_8 b. 34_6 c. 1101_2
2. $\mathrm{C}_{\mathrm{hex}} = 12_{\mathrm{dec}} = 14_{\mathrm{oct}}$
3. What do we know if $P_{n_r} = (x_{n_k}, y_{n_k})$?
4. $7_8 - 4_8$

8.10 SPECIAL CASE—Nonuse of the Subscript Level Indicator: A special rule applies to certain subscripts. The subscript level indicator is <u>not</u> used when <u>all</u> of the following apply:

- the subscript is *numeric;*
- the numeral is a *right subscript to a letter*;
- the subscript is on the *first* level below the baseline of writing.

Example 8.10-1 Apply Definitions i_2 and I_2 .

Each numeral 2 is printed at the subscript level.

When a subscript does not require a subscript indicator, a return to the baseline is implied following the numeric subscript. A baseline indicator is not needed.

⋙ $\mathrm{H_2O}$ 

Letters H and O are on the baseline of writing. Numeral 2 is a subscript.

⋙ $a_1a_2 + b_1b_2 + c_1c_2$ 

All numerals are subscripts. A baseline indicator is not needed to show that the letters and the plus signs are on the baseline level because no subscript indicators are used.

⋙ $\mathrm{Cd(NO_3)_2}$ 

The subscripts are numerals 3 and 2. Everything else is on the baseline of writing, including the closing parenthesis. The second subscript requires a level indicator because it is not a right subscript to a letter.

8.10.1 Further Conditions: The definition of "numeric" and "letter" include the following:

— The numeric subscript may contain a *segmenting comma* or a *decimal point*.

⫸ $x_{1,000}$ ⠭⠂⠠⠴⠴⠴ (The numeral 1,000 is a right subscript to the letter x.)

⫸ $x_{5.3}$ ⠭⠢⠨⠒ (The numeral 5.3 is a right subscript to the letter x.)

— The letter may carry one or more *primes*.

⫸ x'_2 ⠭⠄⠆ (The numeral 2 is a right subscript to x'.)

— The letter may be taken from *any alphabet* in *any typeform*.

⫸ $\aleph_0$ ⠠⠠⠁⠴ (Hebrew aleph, subscript zero)

⫸ $\mathbf{i}_1$ ⠸⠰⠊⠂ (bold English letter i, subscript one.)

— The letter may also carry a *superscript* *(*See **<u>8.16</u>**, *Simultaneous Superscripts and Subscripts*)

— The letter may be part of a *two-letter chemical abbreviation.**

⫸ Cl_4 ⠠⠉⠇⠲ (Two-letter chemical abbreviation for chlorine, subscript four.)

*A one- or two-letter "abbreviation" for a chemical compound follows spacing rules for letters. For example, Fe_2O_3 is brailled without spaces:
⠠⠋⠑⠆⠠⠕⠒

8.10.2 Restrictions: Just because a numeral is a right subscript to a letter does not mean that the special rule can be applied. The subscript level indicator must be used if any of the following conditions apply:

— If the letter is functioning as a numeral in a <u>nondecimal numeration system</u> a subscript indicator is required. (This example shows base 12 numeration.)

⫸ TE_{12} ⠼⠞⠑⠰⠂⠆

— If the letter is part of a <u>word or abbreviation</u> a subscript indicator is required.

⫸ $five_3$ ⠋⠊⠧⠑⠰⠒

— If the subscript contains <u>any symbol other than a numeral</u> with its comma or decimal point a subscript indicator is required.

⫸ x_{2k} ⠭⠰⠆⠅

⫸ $x_{2'}$ ⠭⠰⠆⠄

⫸ x_{-2} ⠭⠰⠤⠆

— If the subscript carries a superscript or subscript of its own a subscript indicator is required.

⋙ x_{2_n} 

— Numeric subscripts on the second or lower levels always require their appropriate subscript level indicators.

⋙ P_{n_1} 

8.10.3 Rewind: Lesson 7 looked at unspaced number/letter combinations. **7.16** illustrated that, when a number appears to the right of a letter on the baseline level, a multipurpose indicator is necessary.

⋙ R12 

Without the multipurpose indicator, the number will read as a subscript.

But if the expression is a subscript, no indicator is needed because the level in effect continues.

⋙ t_{R12} 

8.10.4 Summary: The rules regarding use of the subscript indicator are summarized as follows.

The subscript level indicator must be used with a numeral on the first level below the baseline if the subscript contains any symbol other than a numeral with its comma or decimal point, or if the subscript carries a superscript or subscript of its own. Subscripts on the second or lower levels always require their appropriate subscript level indicators. If the letter is part of a word or an abbreviation, a subscript indicator is required. If the letter is functioning as a numeral in a nondecimal numeration system, a subscript indicator is required.

A subscript level indicator is not used before a numeric subscript on the first level below the baseline of writing when the numeral is a right subscript to a letter. The numeric subscript may contain a segmenting comma or a decimal point. The letter may carry one or more primes or a superscript, may be taken from any alphabet in any typeform, and may be part of a two-letter chemical abbreviation.

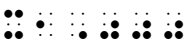
PRACTICE 8G

1) These expressions need subscript indicators in braille: y_{-2}, x_{2+k}, a_{m1}, x_{3_n}, x_{y_2}.
2) These expressions do not use a subscript indicator in braille: x_1, ax_2, CO_2, $z_{.7}$, β_2.
3) Decide whether these expressions require a subscript indicator and braille them correctly: shape_4, Q'_2, $C_6Fe_2O_{12}$, n_k, x_{2k}, $P_{r_{s_t}}$, D_{56}, $G_{9{,}999}$, $2E6B_{16}$.
4) Use $\alpha_1, \beta_1, \gamma_1$ and $\alpha_2, \beta_2, \gamma_2$ to denote the direction vectors $\mathbf{v}_1$ and $\mathbf{v}_2$.
5) *Chemistry*: While Na_2ZnCl_4 could be cooled in the normal way, $Na_2[CoCl_4]$ had to be quenched in the liquid N_2.
6) $f_1(x) = g(x) \cdot q_2(x) + f_2(x)$

8.11 Spaces Within Superscripts and Subscripts: A space usually returns the reader to the baseline. Various strategies are used to retain the level in effect when a space occurs within a superscript or a subscript.

8.11.1 Commas: As noted above, the effect of a level indicator is terminated by a comma unless the comma occurs in a long numeral (a "segmenting" comma). *Recall:* A segmenting comma maintains the level in effect,

⋙ $x_{1{,}000}$



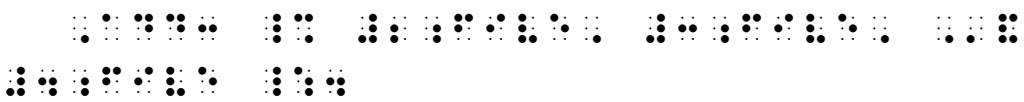
The numeral 1,000 is a right subscript to the letter x.

but a comma followed by a space re-establishes the baseline. The return to the baseline starts at the comma. No baseline indicator is needed.

⋙ x^2, y^2, z^2

⋙ (a_1, a_{1i}, a_{2i})

Example 8.11-1 Add: 2_{five}, 3_{five}, and 4_{five}.



When a comma separates individual items <u>within</u> a superscript or subscript, the contracted comma symbol is used. This symbol represents the comma or the comma and a space. The level in effect continues through this symbol.

Contracted Comma + Optional Space	

⋙ $x^{1,2}$ 

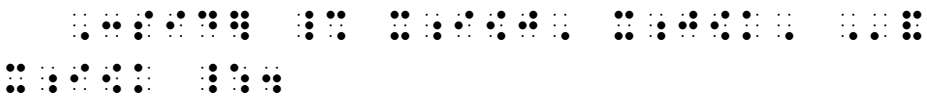
⋙ $x_{i,j,k}$ 

The contracted comma symbol is not to be used to replace a comma on the baseline of writing. A dot 2 or dot 6 comma is used for the comma that is part of the sentence structure.

Example 8.11-2 If ___, then $x_{i,j,k}$, as shown below.

Contracted commas are used within the subscript; a dot 2 comma is used for the comma that is part of the sentence structure (on the baseline level) outside of the Nemeth switches.

Example 8.11-3 Consider $x_{i,j}$, $x_{j,k}$, and $x_{i,k}$.



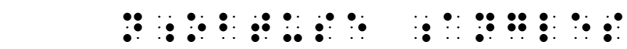
Contracted commas are used within each subscript; a dot 6 comma is used for the commas that are part of the sentence structure (on the baseline level) within the Nemeth switches.

In the next example, the commas separate three terms at the subscript level, but each "n" has a subscript as well. The correct level of each comma is shown by shrewd use of level indicators.

⋙ P_{n_1,n_2,n_3}

8.11.2 Words: If a subscript or a superscript contains a space between words or abbreviations, the level must be restated after the space. The level indicator is unspaced from the following word or abbreviation.

⋙ $n_{\text{obtuse angles}}$



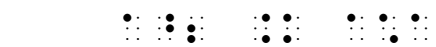
The subscript indicator before the word "angles" shows that the word is in the subscript position.

⋙ $n_{\text{st. angles}}$

The abbreviation st. (for "straight") has a related period. No punctuation indicator is used with abbreviations.

8.11.3 Comparison Signs: The space before a comparison sign returns the reader to the baseline, as illustrated in the next example.

⋙ $a^2 = a \cdot a$



The superscript is 2. The equals sign is on the baseline.

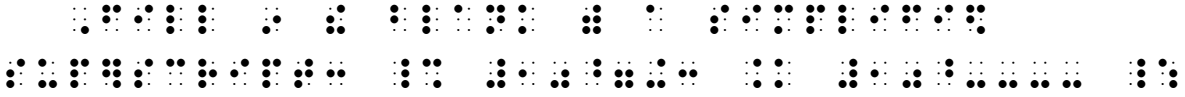
If the comparison sign is within a superscript or a subscript, the level is reinstated before the comparison sign. The indicator is unspaced from the comparison sign. *The space after a comparison symbol preserves the level that is already in effect.*

⋙ $S_{\text{u = a}}$

The subscript is u = a. The level in effect extends through the space following the comparison sign.

8.11.4 Ellipsis or Long Dash in Superscripts and Subscripts: When an ellipsis or long dash is located at a different level from the material preceding it, the appropriate level indicator is used before the ellipsis or long dash. The level indicator is unspaced from the symbol.

Example 8.11-4 Fill in the blank with a simplified superscript: $10^{7+3} = 10^{-}$



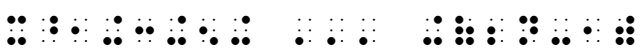
An ellipsis or a long dash within a superscript or a subscript assumes the level in effect.

⋙ $P_{n_1, n_2, \ldots}$

The letter n, the commas, and the ellipsis are at the subscript level. Even though a contracted comma includes the following space, there must be a space before an ellipsis unless it is immediately preceded by an indicator. The subscript level extends through the space before the ellipsis.

The level extends through the required spaces before and after an ellipsis or a long dash.

⋙ $x^{1 + 3 + 5 + \ldots + (2n-1)}$



The superscript is $1 + 3 + 5 + \ldots + (2n - 1)$. *The superscript level extends through the spaces before and after the ellipsis.*

Exception: If an ellipsis or long dash in a superscript or subscript is followed by a sign of comparison or by literary text, reading returns to the baseline level without the need for a baseline indicator.

⋙ $10^{7+}— = 10^{21}$

The first superscript is 7+___. The superscript level extends through the required space before the long dash. The sign of comparison after the dash is assumed to be at the baseline level.

⋙ $10^{7+}—$ equals 10^{21}

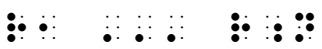
The first superscript is 7+__ . The superscript level extends through the required space before the long dash. The word after the dash is assumed to be at the baseline level.

8.11.5 Ellipsis or Long Dash on the Baseline of Writing: Because the space before an ellipsis or long dash maintains the level in effect, an indicator is required to return to the baseline when the symbol is printed there. The baseline indicator takes the place of the required space.

⋙ $a^1 b^2 c^3 d^4 \ldots z^n$

There is no need to indicate a return to the baseline after a numeric subscript that does not require a subscript indicator.

⋙ $r_1 \ldots r_n$



The ellipsis is printed on the baseline. The subscripts are 1 and n.

8.11.6 Segmented Numbers: The effect of a level indicator extends through the space inserted in a numeral for the purpose of dividing it into short regular groups of digits.

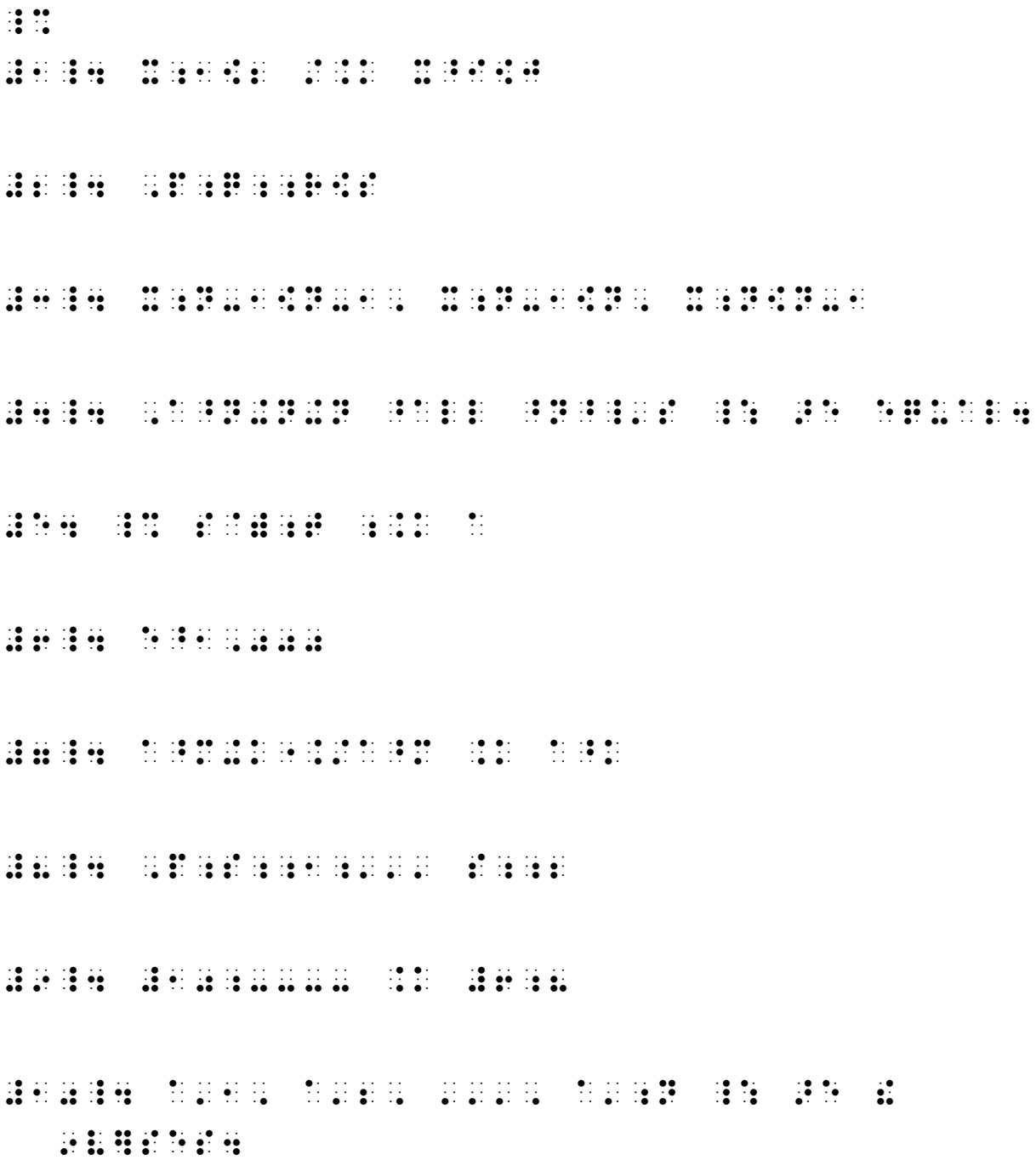
⫸ $e^{3.14159\ 26535}$

⠑⠘⠒⠨⠂⠲⠂⠢⠔ ⠆⠖⠢⠒⠢

The superscript is 3.14159 26535

Instructions: Translate the simulated braille by writing the mathematical characters in the blank space below each line. Show spaces and levels clearly. Compare your translation to the print copy in the answer section.

PRACTICE 8H



More about Superscripts and Subscripts

8.12 Superscript and Subscript Combinations: Combinations of subscripts to superscripts or of superscripts to subscripts require level indicators composed of two or more braille symbols. Keeping in mind that level indicators relate to the baseline, transcribing the following types of expressions is an exercise in logical thinking.

Superscripts with Subscripts

Superscript With Subscript	⠘⠰

⫸ x^{n_1} ⠭⠘⠝⠘⠰⠂

n_1 (n, subscript one) is in the superscript position. "1" is a super-subscript.

⫸ $2^{\mathrm{Y}_0} = \mathrm{Y}_1$ ⠼⠆⠘⠠⠽⠘⠰⠴ ⠨⠅ ⠠⠽⠂

Y_0 (Y, subscript zero) is in the superscript position. "0" is a super-subscript.

Reminder: The subscript indicator is omitted for a numeric subscript to a letter only for subscripts that are located on the *first level below the baseline of writing.* The super/sub indicator is needed to show a numeric subscript in the superscript position.

Subscripts with Superscripts

Subscript With Superscript	⠰⠘

⫸ x_{n^2} ⠭⠰⠝⠰⠘⠆

n^2 (n, superscript two) is in the subscript position. "2" is a sub-superscript.

⫸ P_{3^n} ⠠⠏⠰⠒⠰⠘⠝

3^n (3, superscript n) is in the subscript position. "n" is a sub-superscript.

Reminder: The subscript indicator is required when a numeric subscript to a letter carries a superscript or subscript of its own.

PRACTICE 8I

Instructions: Descriptions are included in a right-hand column for your benefit—do not braille the descriptions. Analyze the levels as you braille and as you proofread.

Superscripts with Subscripts

$2^{\aleph_0} = \aleph_1$	$\aleph_0$ is in the superscript position. (The aleph is <u>not</u> bold.)
$a = 2^{k_1}$ and $b = 2^{k_2}$	k_1 and k_2 are in the superscript position.
$(ab)^x = 2^{k_1 x} \cdot 2^{k_2 x}$	x, $k_1 x$, and $k_2 x$ are superscripts.
$\mathrm{e}^{\mathrm{i}\theta_1}$ times $\mathrm{e}^{\mathrm{i}\theta_2}$ equals $\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}$	$\mathrm{i}\theta_1$, $\mathrm{i}\theta_2$, and $\mathrm{i}(\theta_1 + \theta_2)$ are superscripts.

Subscripts with Superscripts

Z_{5^n}	5^n is in the subscript position.
$7t_{s^4}$	s^4 is in the subscript position.

8.13 Left Subscripts and Superscripts: The appropriate level indicator is brailled before a subscript or superscript printed to the left of its related sign.

⫸ ${}_3\mathrm{x}_1$ ⠰⠒⠐⠭⠂

⫸ ${}_{48}\mathrm{C}_9 \times {}_4\mathrm{C}_4$ ⠰⠲⠦⠐⠠⠉⠔⠈⠡⠰⠲⠐⠠⠉⠲

Reminder: The subscript indicator is not used when a numeral is a right subscript to a letter on the baseline.

⫸ ${}^{14}\mathrm{C}$ ⠘⠂⠲⠐⠠⠉

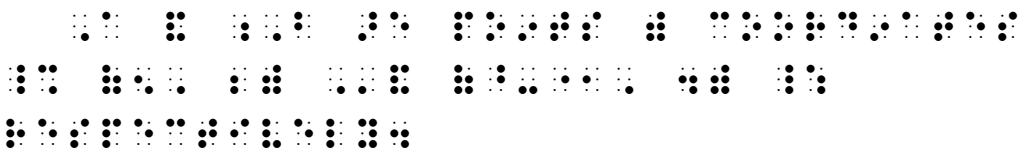
8.13.1 Raised Negative Sign: In some texts, negative numbers are shown with a raised negative sign. The raised position of the negative sign must be shown in braille.

⫸ ${}^{-}4$ ⠘⠤⠐⠲

Example 8.13-1 Explain why $4 + {}^{-}4 = 0$.

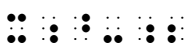
⠠⠑⠭⠏⠇⠁⠔ ⠱⠽ ⠸⠩ ⠼⠲⠬⠘⠤⠐⠲ ⠨⠅ ⠼⠴ ⠸⠱⠲

Example 8.13-2 A and B are points with coordinates (5, 2) and (⁻1, 4) respectively.



Compare the subscripts in the next two examples. The first subscript is −2. A subscript indicator is required because the subscript contains a symbol other than a numeral. In the second subscript, the negative sign is raised—it is a left superscript to the subscript "2".

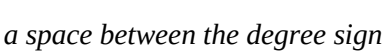
⋙ x_{-2} 

⋙ x_{-2} 

8.13.2 More About the Temperature Abbreviations F (Fahrenheit) and C (Centigrade):
The raised hollow dot representing the degree symbol was introduced in **8.4**. *Review:* When a temperature value is given, the abbreviation "F" or "C" is spaced from the degree symbol in braille, regardless of its print appearance. The next two examples show different spacing in print but the braille transcriptions are identical.

⋙ 98.6° F 

In print, there is a space between the degree sign and the abbreviation "F".

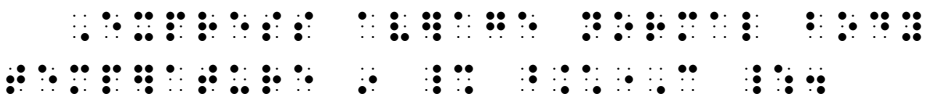
⋙ 98.6°F 

In print, there is no space between the degree sign and the abbreviation "F".

When the temperature abbreviation is shown with the degree symbol and there is no associated value, the letter becomes part of the symbol and is not spaced away from the degree symbol. The "hollow dot" is brailled as a left superscript to a letter. An English letter indicator is not needed because the letter is not standing alone.

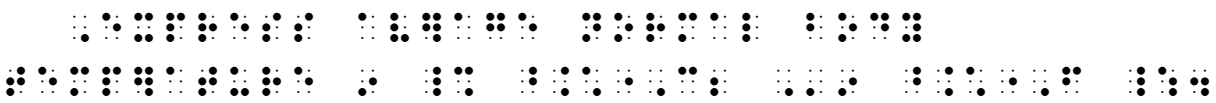
⋙ °C 

Example 8.13-3 Express average normal body temperature in °C.



Punctuation mode is literary.

Example 8.13-4 Express average normal body temperature in °C; in °F.



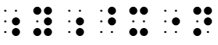
The "C" is followed by a semicolon. No punctuation indicator is used because abbreviations are punctuated in literary mode.

8.14 Further Combinations: The Nemeth codebook illustrates additional combinations of superscripts and subscripts. Due to the obscurity of such complex combinations, only a few examples are shown in this lesson book. Proper interpretation of these characters will require reading the surrounding text in order to apply the correct indicators.

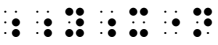
⋙ $_{y}x^{}n$ n is on the baseline of writing and has a left subscript.

Question: Is the subscript y^{x} or $_{y}x$?

This transcription shows left subscript "y" with a sub-superscript "x".



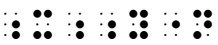
This transcription shows left subscript "x" with a left sub-subscript "y".



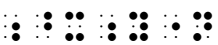
⋙ $_{x_{y}}n$ n is on the baseline of writing and has a left subscript.

Question: Is the subscript x_{y} or ^{x}y ?

This transcription shows left subscript "x" with a sub-subscript "y".



This transcription shows left subscript "y" with a left sub-superscript "x".



8.15 Consecutive Superscripts and Consecutive Subscripts: The appropriate indicator is repeated before a left superscript or a left subscript when two are consecutive—each superscript or subscript has its own indicator. One indicator applies to the expression preceding it and the other to the expression following it.

The space that appears in print helps visually differentiate the two factors, but no space comes between factors in braille unless other spacing rules take precedence.

⫸ $p^{b\ c}q$ ⠏⠘⠃⠘⠉⠐⠟ (There is a space between b and c in print.)

⫸ $P_{x\ y}Q$ ⠠⠏⠰⠭⠰⠽⠐⠠⠟ (There is a space between x and y in print.)

The subscript indicator is or is not brailled according to the rules presented in **8.10**.

⫸ $P_{1\ 2}Q$ ⠠⠏⠂⠰⠆⠐⠠⠟ (There is a space between 1 and 2 in print.)

The first numeric subscript does not require a subscript indicator because it is a right subscript to a letter.

8.16 Simultaneous Superscripts and Subscripts: When a superscript and a subscript are printed directly above and below each other, the subscript is brailled first.

⫸ 10_8^3 ⠼⠂⠴⠰⠦⠘⠒

⫸ ${}^{238}_{92}\mathrm{U}$ ⠰⠔⠆⠘⠆⠒⠦⠐⠠⠥

Example 8.16-1 An example of an I^- iodine anion: ${}^{127}_{53}\mathrm{I}^-$.

⠠⠁⠝ ⠑⠭⠁⠍⠏⠇⠑ ⠷ ⠁⠝ ⠸⠩ ⠠⠊⠘⠤ ⠸⠱ ⠊⠕⠙⠔⠑
⠁⠝⠊⠕⠝⠒ ⠸⠩ ⠰⠢⠒⠘⠂⠆⠶⠐⠠⠊⠘⠤ ⠸⠱⠲

The subscript indicator is or is not brailled according to the rules presented in **8.10**.

⫸ x_1^2 ⠭⠂⠘⠆

The numeric subscript does not require a subscript indicator ***because*** *it is a right subscript to a letter.*

⫸ ${}_m^n x$ ⠰⠍⠘⠝⠐⠭

The subscript is brailled first.

⫸ $\mathrm{n}_{a_{b_c}}^{x^{y^z}}$ ⠝⠰⠁⠰⠰⠃⠰⠰⠰⠉⠘⠭⠘⠘⠽⠘⠘⠘⠵

The entire subscript is brailled before brailling the superscript.

8.17 Nonsimultaneous Superscripts and Subscripts: When a mathematical expression carries both a superscript and a subscript which are not printed directly above and below each other, the superscript and subscript are transcribed *in the same order as in print*, and the baseline indicator is inserted between them.

⫸ $x_1{}^2$ ⠭⠂⠐⠘⠆

⫸ $a^k{}_m$ ⠁⠘⠅⠐⠰⠍

⫸ ${}_y{}^x n$ ⠰⠽⠐⠘⠭⠐⠝

⫸ ${}^b{}_a n$ ⠘⠃⠐⠰⠁⠐⠝

Note: A magnifier and a straightedge can help determine whether superscripts or subscripts are simultaneous or nonsimultaneous. If in doubt whether the expression shows nonsimultaneous super/subscripts or if, instead, the super/subscripts have super/subscripts of their own, compare to the surrounding text for clues.

8.18 Detached Superscripts and Subscripts: When an entire superscript or subscript stands alone, it is brailled without a level indicator. A transcriber's note explains its print position.

Example 8.18-1 In x^2, the ^{2} is the exponent.

⠠⠔ ⠸⠩ ⠭⠘⠆ ⠸⠱⠂ ⠮ ⠼⠃ ⠈⠨⠣⠼⠃ ⠊⠎ ⠗⠁⠊⠎⠫ ⠔
⠏⠗⠔⠞⠈⠨⠜ ⠊⠎ ⠮ ⠑⠭⠏⠕⠝⠢⠞⠲

8.19 Literary Symbols and Level Indicators: If a literary symbol is used mathematically and appears in a superscript or a subscript, its level must be indicated.

Example 8.19-1 A dagger may be used as a superscript in quantum mechanics, for example, $A^{\dagger}$.

⠠⠁ ⠙⠁⠶⠻ ⠍⠁⠽ ⠆ ⠥⠎⠫ ⠵ ⠁ ⠎⠥⠏⠻⠎⠉⠗⠊⠏⠞ ⠔
⠟⠥⠁⠝⠞⠥⠍ ⠍⠑⠡⠁⠝⠊⠉⠎⠂ ⠿ ⠑⠭⠁⠍⠏⠇⠑⠂
⠸⠩ ⠠⠁⠘⠸⠻ ⠸⠱⠲

Summary

Here is a summary of the difference in print among four types of superscripts and subscripts as defined in the Nemeth Code.

- **Consecutive**: A right super/subscript belongs to the preceding character and a left super/subscript belongs to the next character.
- **Simultaneous**: A character has both a superscript and a subscript. The superscript is printed directly above the subscript.
- **Nonsimultaneous**: A character has both a superscript and a subscript. The two scripts are not printed directly above and below each other.
- **Detached**: A super/subscript stands alone without being associated with a letter or number. It is printed slightly above or below the baseline and is smaller than the rest of the text.

PRACTICE 8J

1. Here are some expressions with left superscripts: ${}^{3}x,\ {}^{n}x,\ {}^{-}2 + {}^{-}4 = {}^{-}6,\ ({}^{-}3)^{{}^{-}2+{}^{+}2}$.
2. ${}^{12}_{6}\mathrm{C}$ and ${}^{12}\mathrm{C}$ represent the same carbon isotope.
3. $\mathrm{D}_2\,{}^{18}\mathrm{O}$ is the doubly labeled water isotopologue!
4. In CO_2, the subscript $_2$ means "two oxygen atoms".
5. ${}_{n}P_{r} = K({}_{n-1}P_{r-1})$
6. $a_1^2 + b_1^2 + c_1^2$
7. $[t]_0^4$
8. $2 \times 10_6{}^2 + 3 \times 10_6{}^1 + 2$
9. $P_{xy}Q$
10. $\mathrm{NH_4^+ + Cl^- + H_2O}$
11. *Temperature Conversion:* What is 0 K in °C? in °F? (Answers: –273.15° C; –459.67° F.)

GROUPING SIGNS, cont.

8.20 Review of Rules: The following rules have been discussed in earlier lessons. A review of the cited sections is recommended.

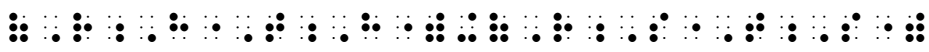
- In mathematical context, enclosure symbols are not considered to be punctuation; they are signs of grouping. Grouping symbols that are part of a mathematical expression must be brailled as Nemeth braille symbols. [3.9]
- The numeric indicator is not used before a numeral which immediately follows an opening sign of grouping unless such numeral is in non-regular type. [3.10, 7.7]
- The English letter indicator is not used when a single English letter or a Roman numeral is entirely enclosed within signs of grouping unless such a letter, or Roman numeral is in non-regular type. This rule does not apply to abbreviations. [4.12.2, 5.3.5]
- Single-letter abbreviations without a related period, or abbreviations whose letters correspond to a shortform without a related period, require an English letter indicator even when enclosed between signs of grouping. [5.19, 5.20]
- When a single English letter, or a Roman numeral is in direct contact with only its opening or only its closing sign of grouping, the English letter indicator is or is not used as though the grouping signs were absent. If the grouping sign carries a prime or other modifying symbol, the English letter indicator is <u>not</u> used. [5.16]

- The appropriate alphabetic indicator must be used with any letter from the German, Greek, Hebrew, or Russian alphabets even when enclosed within, or in contact with, signs of grouping. [5.8.1]
- In mathematical context, non-mathematical symbols of enclosure can be transcribed as Nemeth Code signs of grouping if switching out of Nemeth Code would be cumbersome. [3.13]
- A sequence of mathematical items enclosed in signs of grouping (an "enclosed list") must use the Nemeth braille enclosure symbols. [5.15]
- When square brackets or vertical bars are printed in mathematically-significant boldface, dots 456 are used before the grouping symbol. [7.13.3]

Regarding the main topic in this lesson—superscripts and subscripts—another point can be added.

- When a grouping symbol appears on the baseline level and a level indicator is currently in effect, the baseline indicator is placed <u>before</u> the sign of grouping.

⋙ $(R_H T_H) + (R_S T_S)$



If no subscript indicator is used, a return to the baseline is implied.

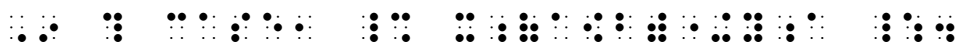
⋙ (a_1, a_2, a_3)

8.21 Grouping Symbols and Level Indicators: Be watchful when determining the level of a grouping sign. Because the bottom margin of grouping signs may extend below the level in effect, it is better to compare the center of the symbol to the surrounding text if its printed level is in question.

Example 8.21-1 Solve $(x^2 + y^2) - (x^2 + y^2)$

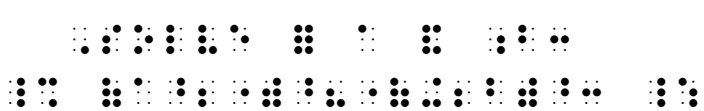
If the grouping symbol starts a different level, the appropriate level indicator is placed before the sign of grouping to properly identify its location.

Example 8.21-2 In this case, $x_{(a,\,b)} + y_a$.



The opening grouping symbol begins the subscript level.

Example 8.21-3 Solve for *a* and *b*: $(a^2)^8(+2b)^3$



All of these grouping symbols are on the baseline of writing.

Review the examples in **<u>8.6.1</u>** with this in mind.

8.22 Grouping Symbols with Super/Subscripts: Lesson 5 taught that when a letter touches only one grouping symbol, the English letter indicator is applied (or is not applied) as though the grouping sign were not present. This rule is illustrated below in the context of a grouping sign that has a subscript and/or superscript.

⫸ $s]_a^b$ *Think:* s_a^b

8.23 More Signs of Grouping: Grouping signs were introduced in **Lesson 3**. Boldface square brackets and vertical bars were shown in **Lesson 7**. The Nemeth Code has devised symbols for eight more grouping signs.

Barred Brackets		
Opening	⟦	
Closing	⟧	
Barred Braces		
Opening	⦃	
Closing	⦄	
Half Brackets		
Upper Left	⌈ or ⌜	
Upper Right	⌉ or ⌝	
Lower Left	⌊ or ⌞	
Lower Right	⌋ or ⌟	

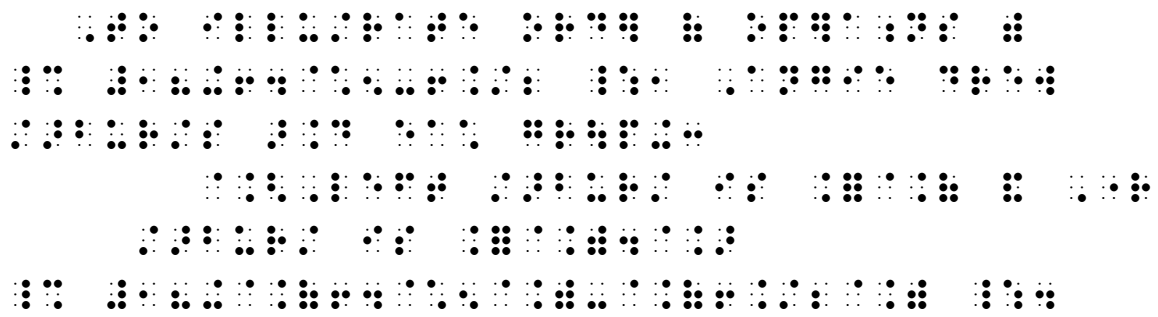
⫸ $A_{n\urcorner\, i}$

⫸ $\llcorner x \lrcorner$

Example 8.23-1 *Integer division* can be defined as $a\backslash b \equiv \lfloor a/b \rfloor$, where "/" denotes normal division and $\lfloor x \rfloor$ is the floor function.

8.24 Transcriber-Devised Grouping Symbols: If a mathematical sign of grouping is not represented in the Nemeth Code, the transcriber is to devise one. Use two or more braille symbols whose last cell is ⣻ for the opening sign and whose last cell is ⣾ for the closing sign. The transcriber-devised symbol should be identified in a transcriber's note or listed on the Special Symbols page at the beginning of the volume according to the guidelines given in *Braille Formats*.

Example 8.24-1 To illustrate order of operations with $18 + 64 \times 5 - 6 \div 2$, Angie drew starbursts around each grouping: $18 + \text{⣻}64 \times 5\text{⣾} - \text{⣻}6 \div 2\text{⣾}$



Instructions: Treat the vertical bar in sentence (7) as an operation sign.

PRACTICE 8K

(1) $\{f_n\}$

(2) $|a_m - a_n|$

(3) $(x_1 y_1 + x_2 y_2)$

(4) $([\mathrm{CH_3}]_2\mathrm{CH})$

(5) $\mathrm{I}_{\mathrm{ue}}^{2"} = (\mathrm{H}'_{44} x'_{\mathrm{ve}})^{+'}$

(6) $\lfloor x \rfloor = m$ if and only if $m \leq x < m + 1$; $\lceil x \rceil = n$ if and only if $n - 1 < x \leq n$.

(7) The dagger and the asterisk are used as superscripts in quantum mechanics: $\mathrm{A}^{\dagger}$, $(x^{\dagger})^{\dagger} = x$, $\langle \phi | \psi \rangle^{*} = \langle \psi | \phi \rangle$.

INTRODUCTION TO DISPLAYED MATHEMATICAL EXPRESSIONS

FORMAT

8.25 Displayed Mathematical Expressions: Up to this point in the lesson material, mathematical expressions in the examples have appeared in line with the narrative. These are referred to as *embedded expressions*. When mathematical material is set apart from the body of the text in the print copy, it is referred to as a *displayed expression*. Various layouts in the print copy are used to set the material apart, for example, skipped lines, centering or other indentation, or off to the side. In braille, margins for displayed mathematical material depend upon the margins of the surrounding text and are transcribed in one of the following formats.

- *In unitemized explanatory portions of the text*, displayed mathematical material begins in cell 3. Runovers begin in cell 5. In other words: text (3-1); displayed material (**3-5**).
- *In itemized text without subdivisions*, displayed mathematical material begins in cell 5. Runovers begin in cell 7. In other words: text (1-3); displayed material (**5-7**).
- *In itemized text with subdivisions*, displayed mathematical material begins in cell 7. Runovers begin in cell 9. These margins apply to both items and subdivisions, to whatever depth. In other words: main division text (1-5); displayed material (**7-9**). Subdivision text (3-5); displayed material (**7-9**).
- *Within or following instructions*, displayed mathematical material begins in cell 5. Runovers begin in cell 7. In other words: instructions (5-3); displayed material (**5-7**).

Notice that in all four layouts, the first cell of the displayed material is indented two cells to the right of the runover cell of the preceding material. These margins apply regardless of the presence of runover material in the preceding paragraph. A line is not skipped above or below displayed mathematical material unless the preceding or following material requires a blank line. [Refer to **8.26** when the context is not mathematical.]

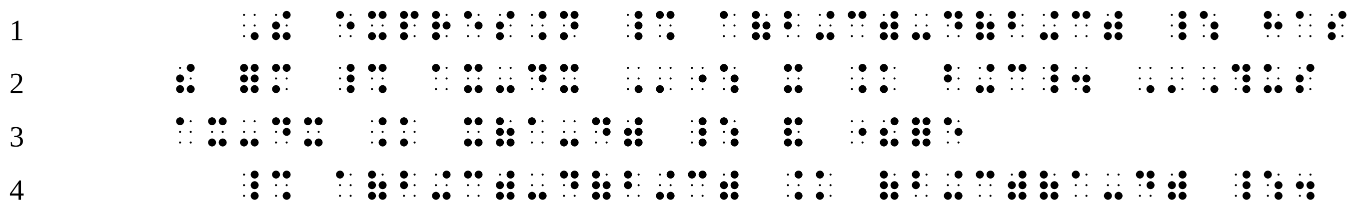
8.25.1 Placement of Code Switch Indicators: When displayed mathematical material is both preceded and followed by UEB text, the expression and the two switch indicators may be placed all together on one line if they will fit within current margins. If more than one line is required for the expression, the opening Nemeth Code indicator is placed at the end of the text line preceding the displayed material. The Nemeth Code terminator is placed at the completion of the displayed expression. In either case, if the indicator will not fit on the current line, it is placed on the following line in the runover position.

— Math displayed to unitemized text: start in cell 3, runover to cell 5.

Example 8.25-1   (Math is displayed to unitemized text.)

The expression $a(b+c)-d(b+c)$ has the form $ax-dx$ where $x=b+c$. Thus $ax-dx=x(a-d)$ and therefore

$$a(b+c)-d(b+c)=(b+c)(a-d).$$



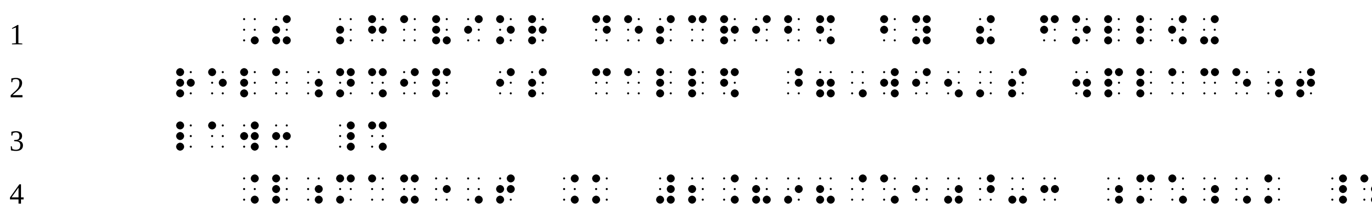
Lines 1-3: Narrative paragraph (3-1) with embedded math.

***Line 4: Because the entire displayed expression will fit on one line with its opening switch indicator and terminator, the open indicator is the first character in the display cell.*

Example 8.25-2   (Math is displayed to unitemized text.)

The behavior described by the following relationship is called **Wien's displacement law:**

$$\lambda_{\text{max}}T = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K}$$



Lines 1-3: Narrative paragraph (3-1).

Line 3: The opening Nemeth Code indicator is placed at the end of the line of text preceding the displayed expression. (Reminder: The switch to Nemeth Code terminates the UEB bold passage.)

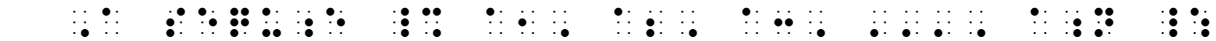
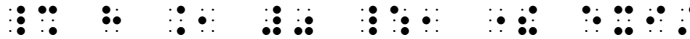
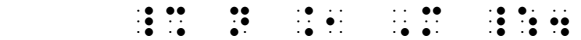
***Line 4: Displayed to unitemized text is placed in cell 3. The Nemeth Code terminator follows the end of the math expression.*

Example 8.25-3 (Math is displayed to unitemized text.)

A sequence $a_1, a_2, a_3, \ldots, a_n$ is said to *converge* if, for each $h > 0$, there exists

a positive number M such that $|a_n - A| < h$, for all $n > M$.

A sequence that does not converge is said to *diverge*.

1 
2 
3 
4 
5 
6 
7 
8 

Lines 1-3: Narrative paragraph (3-1) with embedded math.

***Lines 4-6: The displayed math expression begins in cell 3, with runovers in cell 5.*

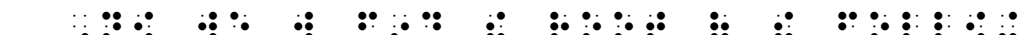
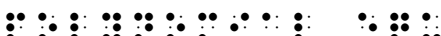
Lines 7-8: Continuation of narrative paragraph.

Example 8.25-4 (Math is displayed to unitemized text.)

Now we will find the root of the following polynomial equation:

$$x^5 + x^4 - 18x^3 + 4x^2 + 88x - 96 = 0$$

How many roots do you predict we will find?

1 
2 
3 
4 
5 

Lines 1-2, 5: Narrative paragraph begins (3-1) and continues on line 5 in runover cell (1).

***Lines 3-4: Displayed math begins in cell 3. Since both switch indicators do not fit on this line, the opening Nemeth Code indicator is on line 2 (in the narrative paragraph), and the Nemeth Code terminator falls in the runover position of the current text – cell 5.*

Line 5: Continuation of 3-1 paragraph (cell 1).

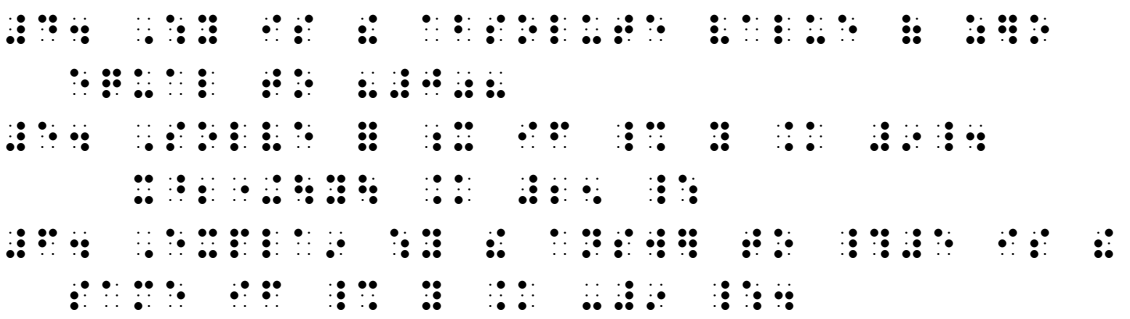
— Math displayed to itemized text: start in cell 5, runover to cell 7.

Example 8.25-5 (Math is displayed to itemized text.)

4. Why is the absolute value of zero equal to "0"?
5. Solve for x if y = 9.

$$x^2 + |y| = 25$$

6. Explain why the answer to #5 is the same if y = –9.



Lines 1-2: Itemized material with no subdivisions (1-3).

Line 3: New item begins in cell 1.

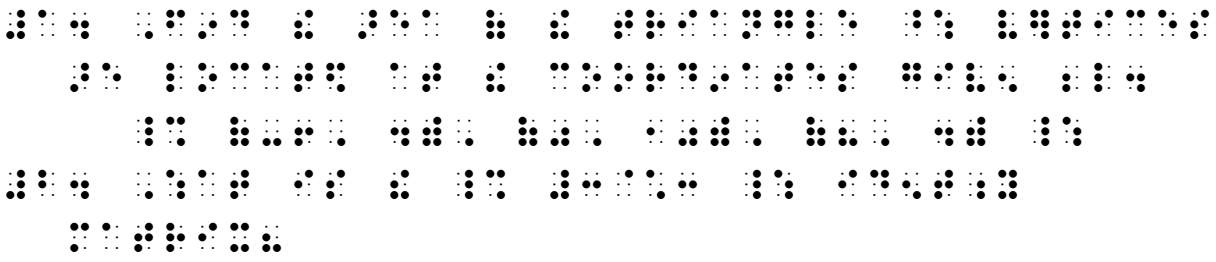
***Line 4: The displayed item begins in cell 5 even though the text to which it applies does not have a runover.*

Lines 5-6: Itemized material continues (1-3).

Example 8.25-6 (Math is displayed to itemized text.)

1. Find the area of the triangle whose vertices are located at the coordinates given below.

   (-6, 4), (0, 10), (8, 4)
2. What is the 3 x 3 identity matrix?



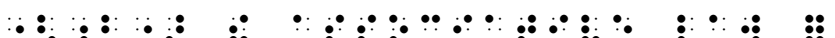
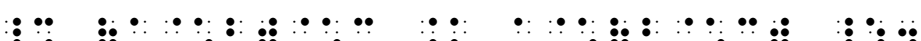
Lines 1-2 and 4-5: Itemized material with no subdivisions (1-3).

***Line 3: Cell 5 is the starting cell for math displayed to itemized text.*

— Math displayed to subitems: start in cell 7, runover to cell 9.

Example 8.25-7 (Math is displayed to subitems.)

5. Give two examples illustrating
   (a) the associative law for addition
   $$(a + b) + c = a + (b + c).$$
   (b) the associative law for multiplication
   $$(a \times b) \times c = a \times (b \times c).$$

1 
2 
3 
4 
5
6 

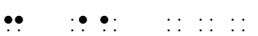
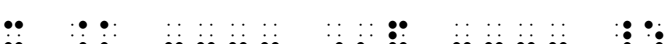
Line 1: Main item begins in cell 1.

Lines 2 and 4-5: Each subitem begins in cell 3. Line 5 shows a runovers in cell 5.

***Lines 3 and 6: Displayed math is in cell 7, which is two cells in from the runover of subitems, whether or not runovers occur.*

Example 8.25-8 (Math is displayed to subitems.)

2. Now solve each of the following equations.
   a. $3(x + 5) = 6x + 6$ b. $x^2 - 25 = 0$
   $x =$ ____ $x =$ ____ and ____

1 
2 
3 
4 
5 
6 

Lines 1-2: Main item begins in cell 1. The opening Nemeth Code indicator is placed at the end of the line of text preceding the math items. Because there is no room for the switch indicator on line 1, it falls in the runover position of the current text (cell 5).

Lines 3 and 5: Each subitem begins in cell 3, not side-by-side as printed.

***Lines 4 and 6: Displayed math begins in cell 7, which is two cells in from the runover of subitems, whether or not runovers occur. (Subitems are 3-5.)*

Line 6: Nemeth Code is terminated at the end of the displayed material.

8.25.2 Look For Context Clues: Each separate expression should start on a new line in the main display cell, even if printed widely-spaced on one line. Notice paragraph patterns in the print copy to determine whether the narrative following displayed material is a continuation of the preceding text or if it is the start of a new paragraph.

Example 8.25-9 (Math is displayed to unitemized text.)

A sentence with an equal sign is called an **equation**. Here are three examples.

$$27 = 9(4 - 1) \qquad 1 + 1 = 3 \qquad x + 7 = 50$$

Which equation is true? Which equation is false? Which equation may be either true or false?

```
1     ⠠⠁ ⠎⠢⠞⠰⠑ ⠾ ⠁⠝ ⠑⠟⠥⠁⠇ ⠎⠊⠛⠝ ⠊⠎ ⠉⠁⠇⠇⠫ ⠁⠝
2   ⠘⠂⠑⠟⠥⠁⠰⠝⠲ ⠠⠐⠓ ⠜⠑ ⠹⠗⠑⠑ ⠑⠭⠁⠍⠏⠇⠑⠎⠲ ⠸⠩
3     ⠼⠆⠶ ⠨⠅ ⠼⠔⠷⠲⠤⠂⠾
4     ⠼⠂⠬⠂ ⠨⠅ ⠼⠒
5     ⠭⠬⠶ ⠨⠅ ⠼⠢⠴ ⠸⠱
6   ⠠⠱ ⠑⠟⠥⠁⠰⠝ ⠊⠎ ⠞⠗⠥⠑⠦ ⠠⠱ ⠑⠟⠥⠁⠰⠝ ⠊⠎ ⠋⠁⠇⠎⠑⠦
7   ⠠⠱ ⠑⠟⠥⠁⠰⠝ ⠍⠁⠽ ⠆ ⠑⠊ ⠞⠗⠥⠑ ⠕⠗ ⠋⠁⠇⠎⠑⠦
```

Lines 1-2: Narrative (3-1).

***Lines 3-5: Three separate displayed expressions, each in cell 3. (In print, the three equations are on one line with extra spacing between them.)*

Lines 6-7: Continuation of narrative paragraph, in the runover cell (cell 1).

When punctuation is used to separate displayed items, the items may continue on the same line, running over as necessary in the appropriate runover cell.

Example 8.25-10 (Math is displayed to itemized text.)

21. Use a number line to show the following inequalities:

$$-6 < -5, \quad 0 < +6, \quad -8 < +2, \quad -1 > -5$$

22. ...

```
1 ⠼⠃⠁⠲ ⠠⠥⠎⠑ ⠁ ⠝⠥⠍⠃⠻ ⠇⠔⠑ ⠞⠕ ⠩⠪ ⠮ ⠋⠕⠇⠇⠪⠬
2   ⠔⠑⠟⠥⠁⠇⠊⠞⠊⠑⠎⠒ ⠸⠩
3     ⠤⠼⠖ ⠐⠅ ⠤⠼⠢⠠ ⠼⠴ ⠐⠅ ⠬⠖⠠ ⠤⠼⠦ ⠐⠅ ⠬⠆⠠
4       ⠤⠼⠂ ⠨⠂ ⠤⠼⠢ ⠸⠱
5 ⠼⠃⠃⠲ ⠲⠲⠲
```

Lines 1-2 and 5: Itemized material with no subdivisions (1-3).

***Lines 3-4: The displayed math begins in cell 5 and continues in the runover cell (cell 7). (In print, the inequalities are separated by commas and are all printed on one line.)*

When sentence-structure punctuation is included in the displayed math, each expression may begin in the primary display cell if doing so more closely matches the print layout.

Example 8.25-11 (Two math items are displayed to unitemized text.)

The result of nested floor or ceiling functions is the innermost function:

$$\left\lfloor \lceil x \rceil \right\rfloor = \lceil x \rceil,$$
$$\left\lceil \lfloor x \rfloor \right\rceil = \lfloor x \rfloor.$$

```
1   ⠠⠮ ⠗⠑⠎⠥⠇⠞ ⠷ ⠝⠑⠌⠫ ⠋⠇⠕⠕⠗ ⠕⠗ ⠉⠑⠊⠇⠬
2 ⠋⠥⠝⠉⠰⠝⠎ ⠊⠎ ⠮ ⠔⠝⠻⠍⠕⠌ ⠋⠥⠝⠉⠰⠝⠒ ⠸⠩
3   ⠈⠰⠷⠈⠘⠷⠭⠈⠘⠾⠈⠰⠾ ⠨⠅ ⠈⠘⠷⠭⠈⠘⠾⠠
4   ⠈⠘⠷⠈⠰⠷⠭⠈⠰⠾⠈⠘⠾ ⠨⠅ ⠈⠰⠷⠭⠈⠰⠾ ⠸⠱⠲
```

Lines 1-2: Narrative paragraph (3-1).

***Line 3: Math displayed to unitemized text begins in cell 3.*

***Line 4: New displayed math item also begins in cell 3.*

8.25.3 Instructions with Displayed Material: If displayed mathematical material appears within or immediately following instructions, the displayed material begins in cell 5 with runovers in cell 7. Instructional text (5-3): displayed material (**5-7**).

— Math displayed to instructions: start in cell 5, runover to cell 7.

Example 8.25-12 (Math is displayed to instructions.)

Use the equation to the right to answer each question. $x + y + z = \$25{,}000$

1. If x = $5,000, express y in terms of z.
2. If y = $10,000, express z in terms of x.
3. If z = $15,000, express x in terms of y.

1
2
3
4
5
6
7
8
9

Lines 1-2: According to UEB and Braille Formats, boldface print used for instructions is disregarded in braille. 5-3 margins characterize instructions in Nemeth Code.

***Line 3: Displayed math begins two cells in from the runover cell of the previous text. The two switch indicators fit on the same line as the displayed math.*

Lines 4-9: Itemized problems (1-3).

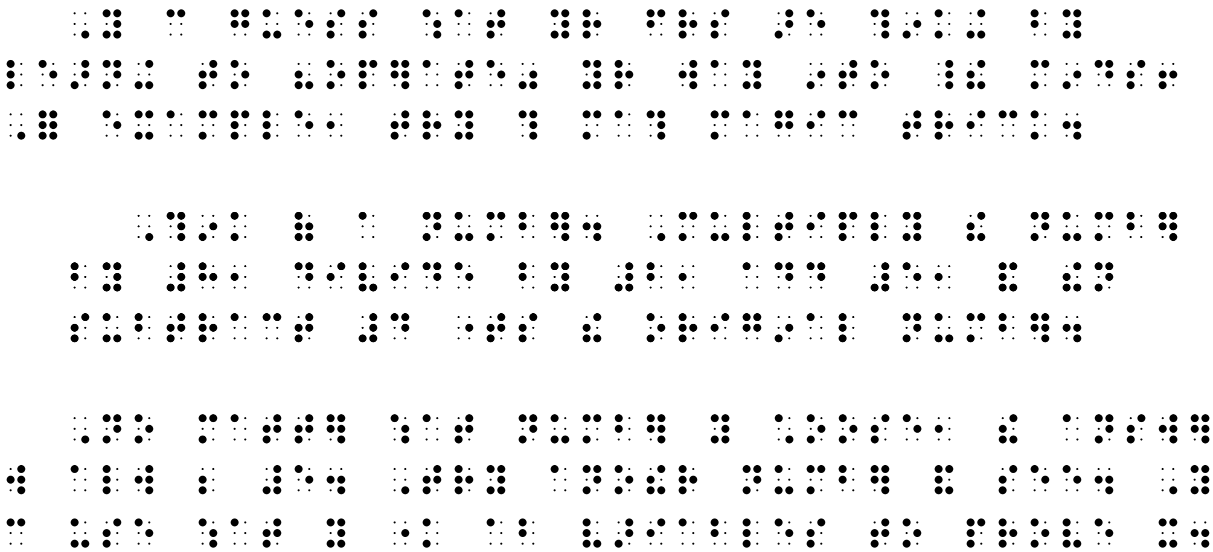
8.26 Displayed Narrative Material: Displayed literary text should follow the guidelines in *Braille Formats* with the exception that blocked paragraphs are not used throughout a technical document. If it is difficult to determine whether the material is mathematical or narrative, use your best judgement and apply consistent formatting throughout the transcription.

Example 8.26-1

You can guess what your friends are thinking by learning to "operate" your way into their minds! For example, try this math magic trick.

> **Think of a number. Multiply the number by 8, divide by 2, add 5, and then subtract 4 times the original number.**

No matter what number you choose, the answer will always be 5. Try another number and see. You can use what you know about variables to prove it.



Lines 1, 5, and 9: According to Nemeth Code rules, the first line of each paragraph is indented two cells from the paragraph's left margin.

Lines 4 and 8: According to Braille Formats *guidelines, displayed literary material is preceded and followed by a blank line.*

Lines 5-7: According to Braille Formats *guidelines, cell 3 is the adjusted left margin in narrative.*

PRACTICE 8L

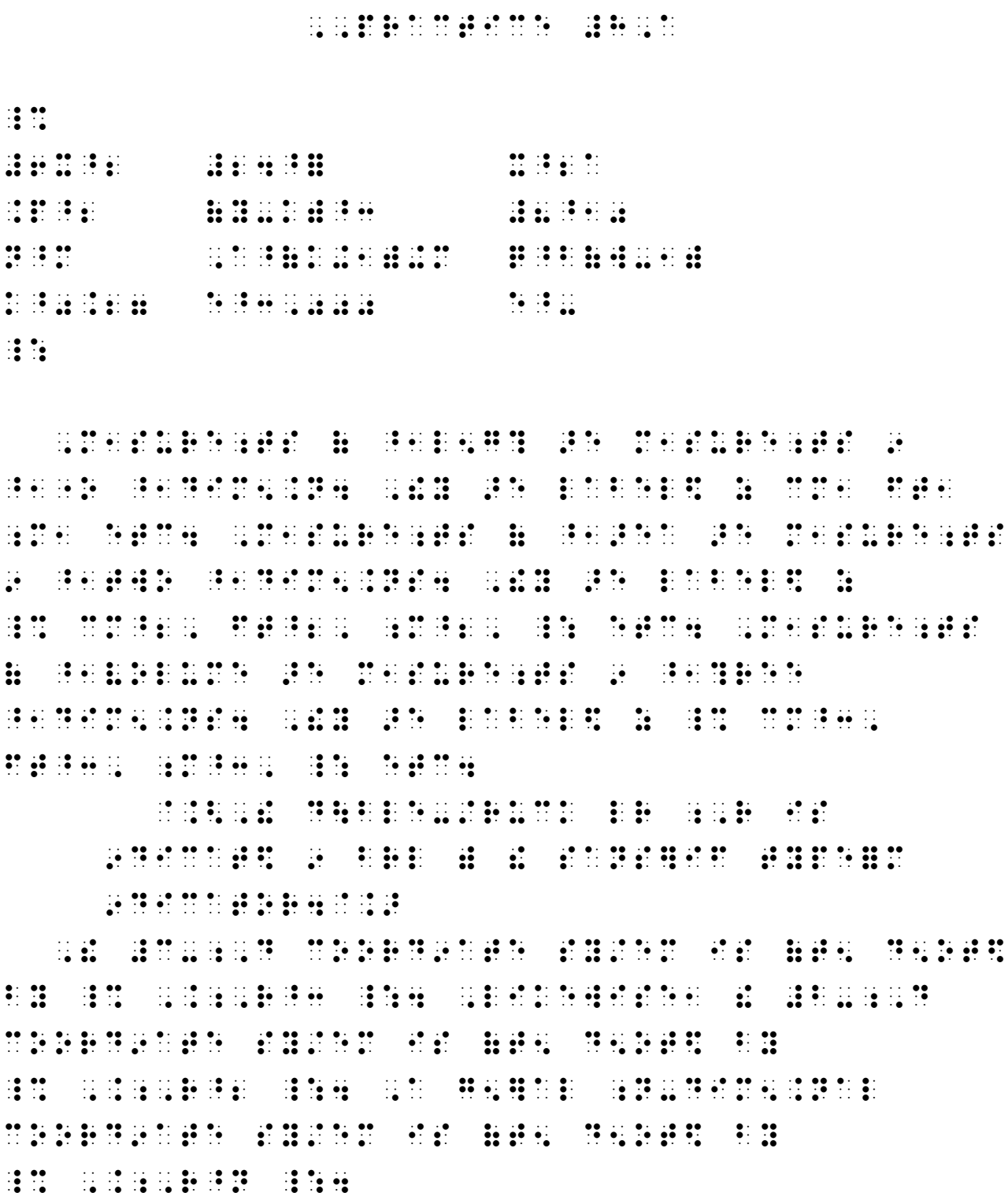
Solve this polynomial using basic algebra. *Hint:* First factor out "x" to make it a quadratic equation.

$$x^3 + 2x^2 - x = x(x^2 + 2x - 1)$$

Do you notice a familiar pattern?

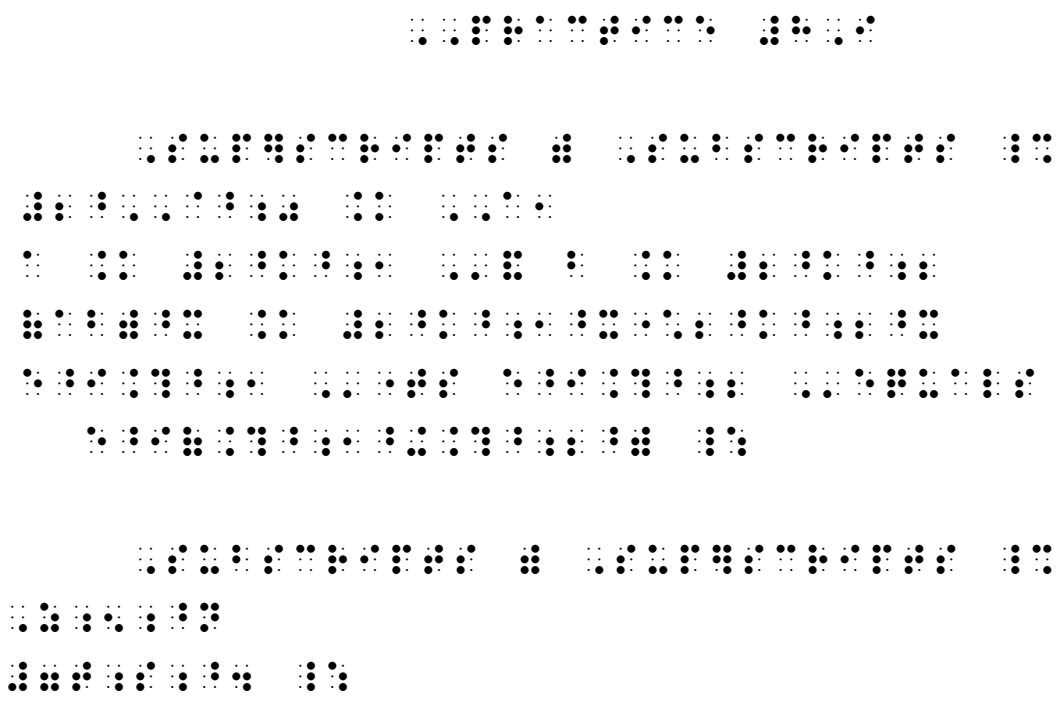
For further practice, see Appendix A—Reading Practice.

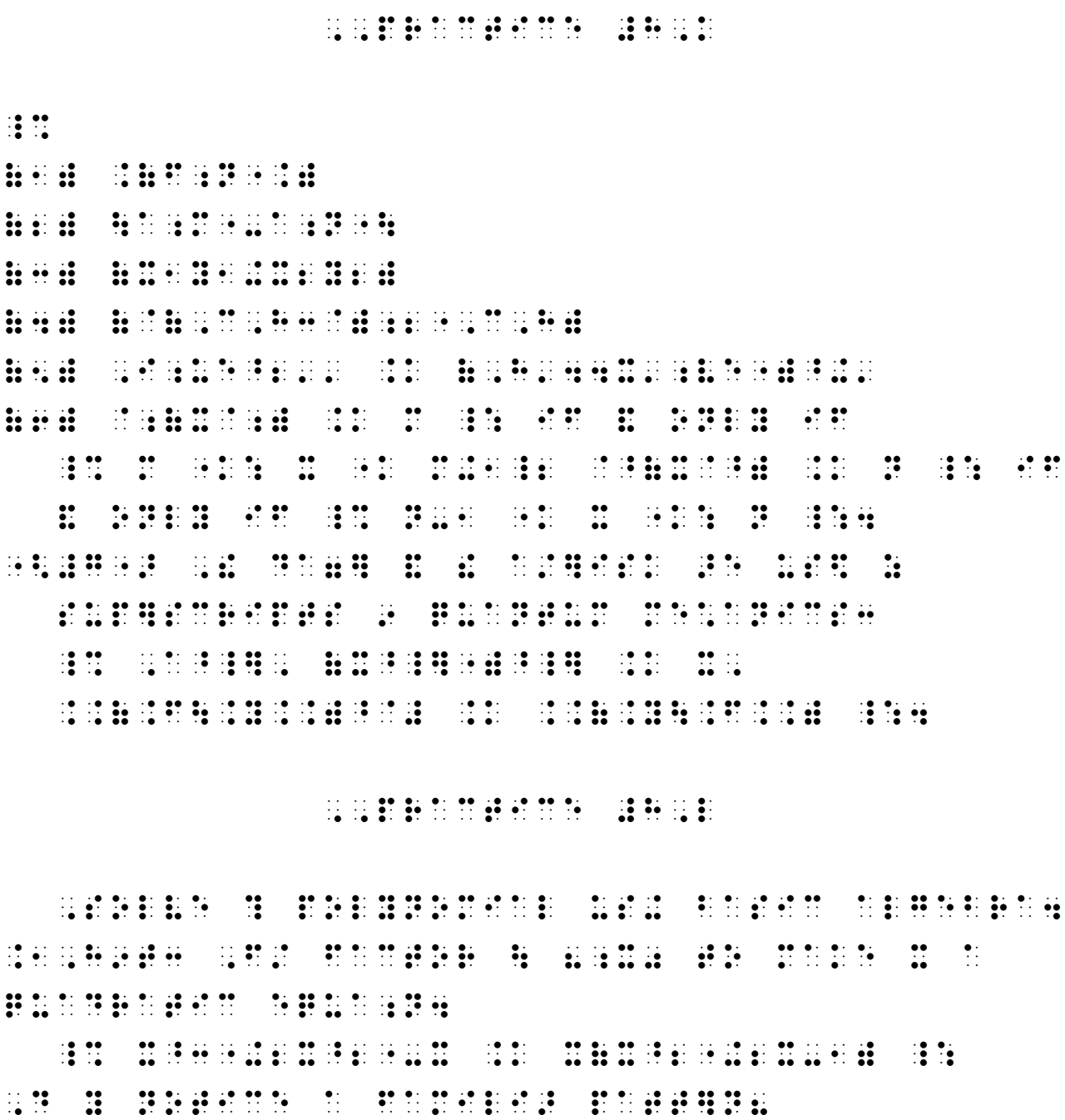
ANSWERS TO PRACTICE MATERIAL



PRACTICE 8H

1. $x_{1,\,2} \neq x^{i,\,j}$
2. $P_{q_{r,\,s}}$
3. $x_{n-1,\,n-1}, x_{n-1,\,n}, x_{n,\,n-1}$
4. $\mathrm{A}^{\mathrm{n+n+n\ all\ n's}}$ are equal.
5. $s]_{t\,=\,a}$
6. $e^{1{,}000}$
7. $a^{m+k} \div a^{m} = a^{k}$
8. $P_{s_1 \ldots\, s_2}$
9. $10\_ = 6_8$
10. $a'_1, a'_2, \ldots, a'_n$ are the inverses.





EXERCISE 8

Exercise 8 will be available when this course is finished being written and is no longer "Provisional". Proceed to Lesson 9.