

Braille observations: Each equation begins in the same cell because the terms are not vertically aligned in print. A blank line is inserted before and after the system. Numeric indicators are used as needed according to rules of the Nemeth Code for nonspatial material. Code switch indicators do not interfere with the arrangement. The Nemeth Code terminator is placed in cell 1 after a blank line following the spatial arrangement.

Note: The Nemeth Code calls such an arrangement an "ununified system of equations" because the equations are not joined by a grouping symbol.

Example 16.2-2 Using three equations, we can solve for x, y, and z:

$$\begin{aligned} 4x - 3y + z &= -10 \\ 2x + y + 3z &= 0 \\ -x + 2y - 5z &= 17 \end{aligned}$$

Print observation: The terms on both sides of the equals signs are vertically aligned.

Braille observations: A blank line is inserted before and after the system. Spaces are inserted in order to maintain alignment as printed. Numeric indicators used as needed according to rules of the Nemeth Code for nonspatial material.

The BANA Nemeth Code Technical Committee is discussing details regarding the situation encountered below. A provisional solution is provided.

When numerical coefficients with a different number of digits need to line up by place value in order to follow print alignment, a numeric indicator is not inserted in the space.

Example 16.2-3

$$\begin{aligned} 4x - 13y &= -10 \\ -x + 2y &= 17 \end{aligned}$$

A space is inserted before the single digit "2" to achieve alignment with the "13" above it.

16.6 Enlarged Parentheses: Notice that the enlarged braille parentheses are formed by inserting a dot 6 before the ⠠ or the ⠡ symbol of the normal parenthesis symbol.

| Enlarged Parentheses | | | |
|----------------------|---|--------|------------------------|
| Left | (| ⠠⠠⠠⠠⠠⠠ | (on two or more lines) |
| Right |) | ⠠⠠⠠⠠⠠⠠ | (on two or more lines) |

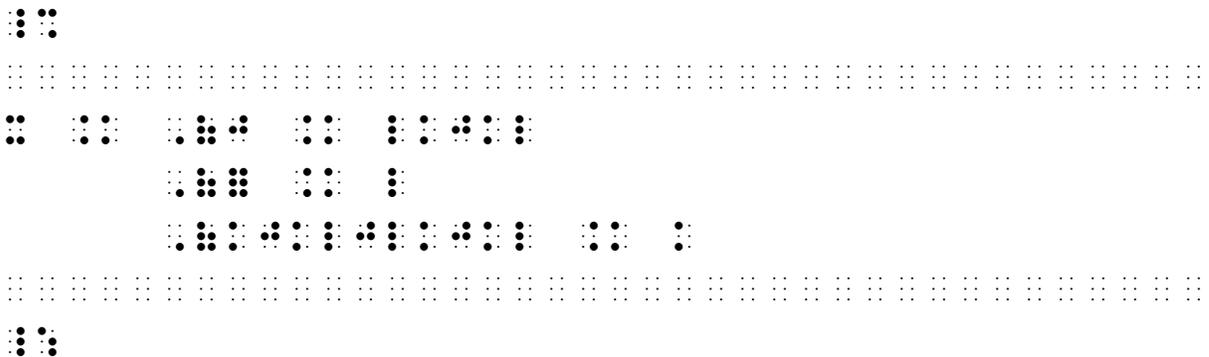
16.7 Placement of Symbols: Symbols which appear outside of the enlarged grouping symbol and which apply to the arrangement are placed on the top line of the arrangement, even if the items are centered in print. Also note the following details:

- The English letter indicator is (or is not) used according to rules of the Nemeth Code for nonspatial material.
- Alignment is maintained only if all terms and symbols are aligned in print.

Example 16.7-1

$$x = \left(\begin{array}{l} j = lkjkl \\ = l \\ kjkljljkjl = k \end{array} \right)$$

Print observations: The anchor is centered with respect to the enlarged grouping symbol to its right. The three-line link is grouped with an enlarged left parenthesis. Equals signs in the grouped arrangement are aligned but terms are not.



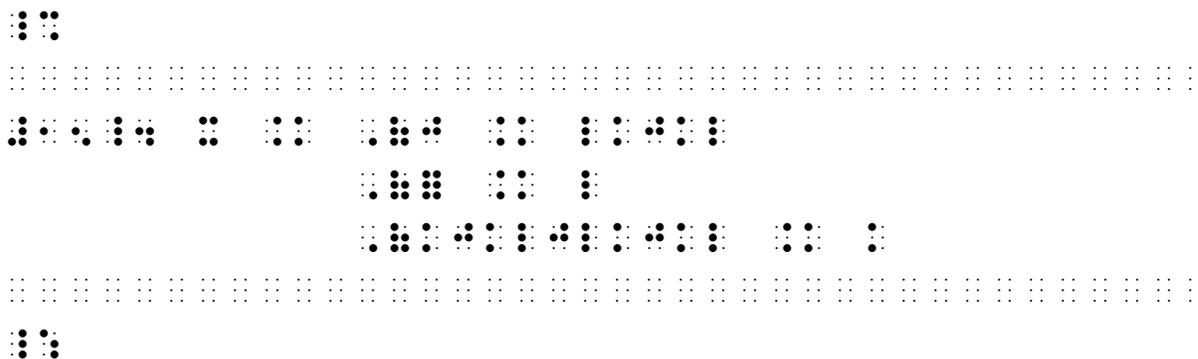
Braille observations: The anchor is aligned with the top line of the arrangement. Each enlarged grouping symbol begins in the same cell. The grouped items are left-adjusted because the terms are not aligned. A general omission symbol indicates the blank space before the middle equals sign. Code switch indicators do not interfere with the spatial arrangement.

16.8 Placement of Identifiers and Punctuation: Identifiers and punctuation which appear outside of the enlarged grouping symbol and which apply to the arrangement are placed on the top line of the arrangement, even if the items are centered in print. Here is the same example, now identified with an item number "15."

Example 16.8-1

$$15. \quad x = \left(\begin{array}{l} j = lkjkl \\ = l \\ kjkljlkjkl = k \end{array} \right.$$

Print observation: The item number and the anchor are centered with respect to the associated enlarged grouping symbol to the right.

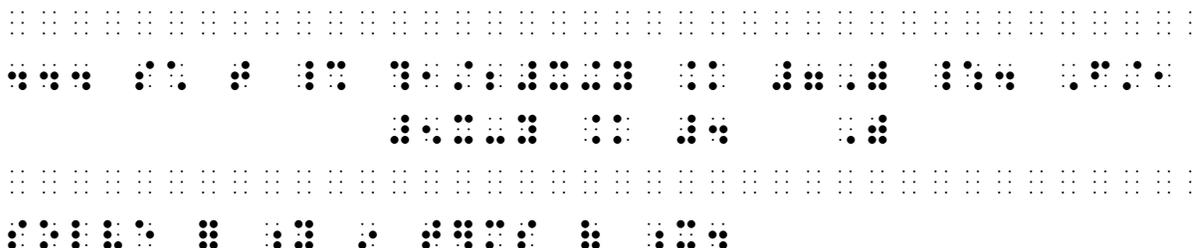


Braille observation: Item number and anchor are aligned with the top (first) line of the arrangement.

Example 16.8-2

... such that $\left. \begin{array}{l} \frac{1}{2}x + y = 7 \\ 5x - y = 4 \end{array} \right\} \text{. First, solve for } y \text{ in terms of } x.$

Print observations: A vertical arrangement is embedded within the narrative. It is grouped on the right with an enlarged right parenthesis. A period follows enlarged parenthesis, centered to the arrangement.



Braille observations: The narrative sentence ends with a displayed expression. The preceding text and the following period are brailled on the top line of the arrangement even though they are centered to the expression in print. A blank line is inserted before and after the lines with the embedded array.

PRACTICE 16D

Find x and y in terms of a and b .

1. $\begin{cases} x + y = 0 \\ x + ay = 1 \end{cases} \quad (a \neq 1)$

2. $\begin{cases} ax + by = 0 \\ x + y = 1 \end{cases} \quad (a \neq b)$

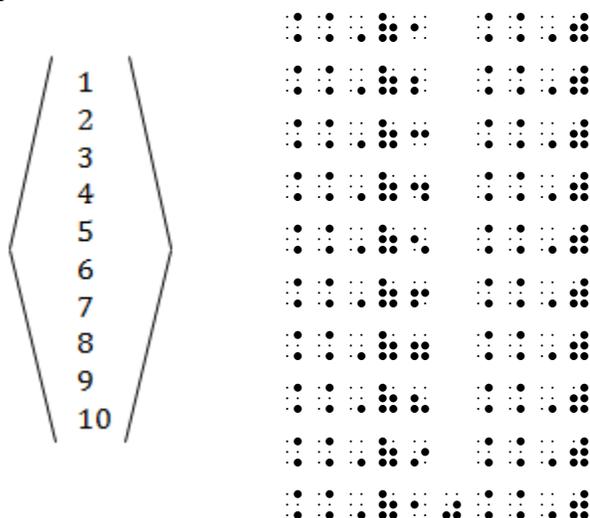
3. $\begin{cases} ax + by = 0 \\ a^2x + b^2y = 1 \end{cases} \quad (a \neq 0, b \neq 0, a \neq b)$

4. $\left. \begin{array}{l} a = \frac{x+y}{x-y} \\ b = \frac{x-y}{x+y} \end{array} \right) -1 < x < 1, -1 < y < 1$

16.11 More Enlarged Signs of Grouping: In addition to the enlarged braces and parentheses, the Nemeth Code provides symbols for six other enlarged signs of grouping. Notice that each enlarged braille symbol is formed by inserting a dot 6 before the $\left[\right]$, $\langle \rangle$, or $\{ \}$ symbol of the normal symbol.

| | | | |
|---------------------------------|------------------|------------------|--------------------------|
| Enlarged Vertical Bar | | | |
| Single | | $\left[\right]$ | $\left[\right]$ |
| Double | | $\left[\right]$ | $\left[\right]$ |
| Enlarged Brackets | | | |
| Left | [| $\left[\right]$ | Right] $\left[\right]$ |
| Enlarged Angle Brackets | | | |
| Left | < | $\left[\right]$ | Right > $\left[\right]$ |
| Enlarged Barred Brackets | | | |
| Left | $\left[\right]$ | $\left[\right]$ | Right $\left[\right]$ |
| Enlarged Angle Braces | | | |
| Left | $\left[\right]$ | $\left[\right]$ | Right $\left[\right]$ |
| Enlarged Half Brackets | | | |
| Upper Left | $\left[\right]$ | Upper Right | $\left[\right]$ |
| Lower Left | $\left[\right]$ | Lower Right | $\left[\right]$ |

Example 16.11-1



DETERMINANTS AND MATRICES

16.12 Definition and Recognition: *Determinants* and *matrices* (singular: *matrix*) are arrangements of items in rows and columns which are enclosed between left and right grouping symbols. Items can be numbers, symbols, or mathematical expressions.

Here is a 2×2 ("two by two") determinant enclosed between enlarged vertical bars.

$$\left| \begin{array}{cc} a & b \\ c & d \end{array} \right|$$

Here is a 2×3 ("two by three") matrix enclosed between enlarged brackets.

$$\left[\begin{array}{ccc} 1 & 9 & -13 \\ 20 & 5 & -6 \end{array} \right]$$

Determinants and matrices may also be referred to as "arrays." An array can be composed of only one column or only one row. These will be studied at the end of this section.

16.13 Transcription Rules for Determinants and Matrices: Observe the following rules regarding the transcription of the two examples shown above.

16.13.1 Blank Lines: Determinants and matrices are spatial arrangements. Thus, a blank line is to be left above and below each array. If the arrangement begins at the top of a braille page, it may begin on line 1 provided no running head is in use; if the arrangement ends at the bottom of a braille page, it may end on line 25. In either case, the rightmost symbol of the arrangement must not fall within three cells of the page number. (See **10.31** in Lesson 10 for details regarding layout at the top or bottom of a page.)

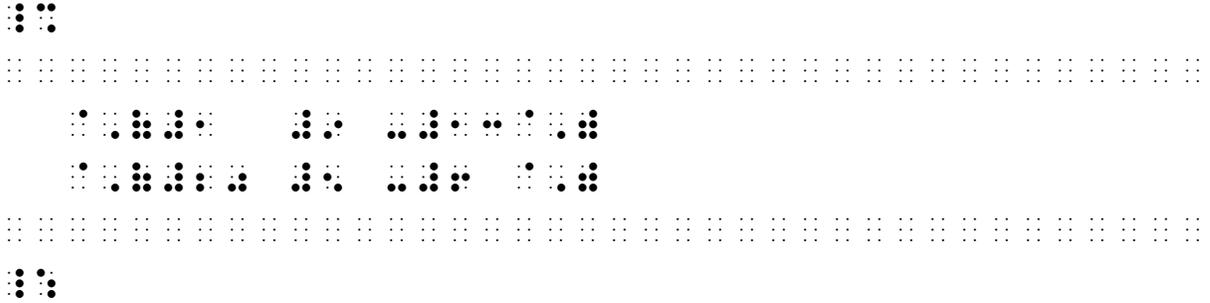
16.13.2 Grouping Symbols: For an array consisting of two or more rows, enlarged grouping symbols are used on each braille line. Grouping symbols are vertically aligned.

16.13.3 Placement of Items: Each entry is moved as far left as possible in its column. Consequently, each opening sign of grouping will be in direct contact with the first entry of each row in the array. Centering and other forms of alignment are not permitted. One column of blank cells is left between the columns of the arrangement. That is, one blank cell separates the widest entry in a column from the beginning of the next column. At least one closing sign of grouping must be in direct contact with an entry in the array. The widest entry in the last column determines the placement of the right (closing) enlarged grouping signs.

16.13.4 Numeric and Letter Indicators: The numeric indicator is used with numeric entries in an array, even when such entries are in direct contact with an opening grouping sign.

Example 16.13-1 $\left[\begin{array}{ccc} 1 & 9 & -13 \\ 20 & 5 & -6 \end{array} \right]$

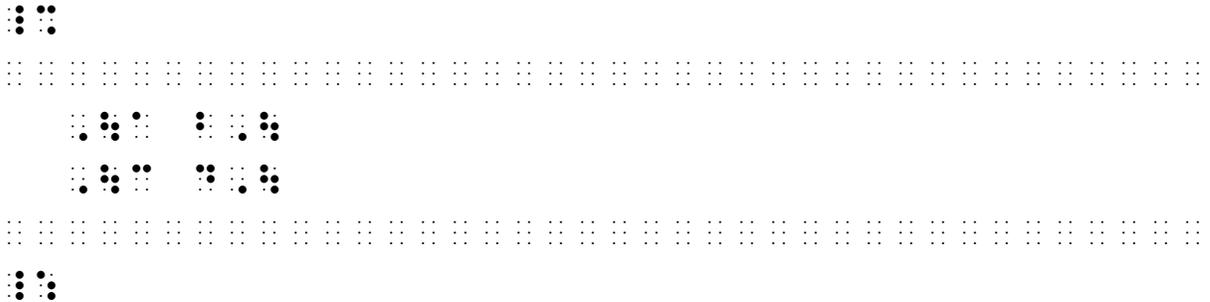
Print observation: The numerical entries are centered in their columns.



Braille observations: The numeric indicator is used for each number. Items begin in the same cell in each column (entries are left-aligned within each column). One blank cell separates the widest entry in a column from the beginning of the next column. The widest entry in the last column determines the placement of the right (closing) enlarged grouping signs. Enlarged brackets enclose this two-row array.

The English letter indicator is not used with any letter or combination of letters in an array.

Example 16.13-2 $\left| \begin{array}{cc} a & b \\ c & d \end{array} \right|$



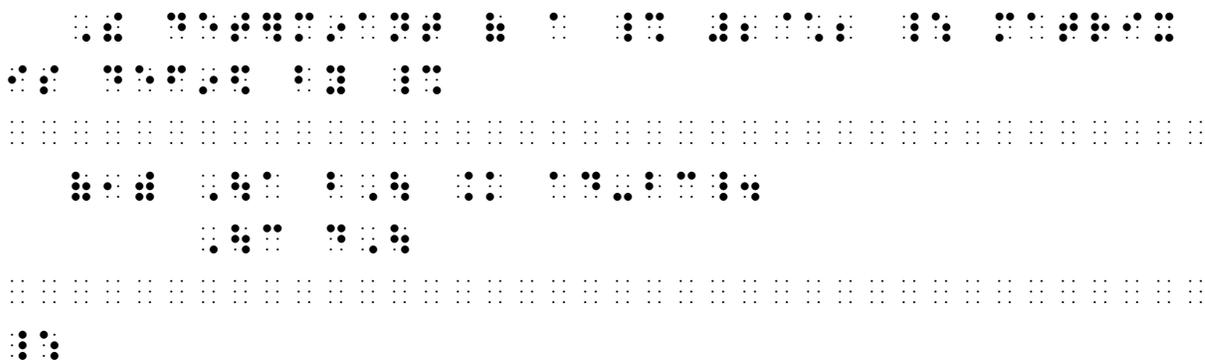
Each English letter is brailled without a letter indicator. Enlarged vertical bars enclose this two-row array.

16.13.5 Placement of Identifiers, Symbols, and Punctuation: Material outside of an array (such as identifiers, punctuation, signs of operation, or signs of comparison) are placed on the top line of the arrangement even though the material may be centered in print.

Example 16.13-3 The determinant of a 2×2 matrix is defined by

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc. \quad (1)$$

Print observations: An equals sign and expression are centered to the array. A period ends the statement. The expression is identified to the right with a numeral in parentheses.

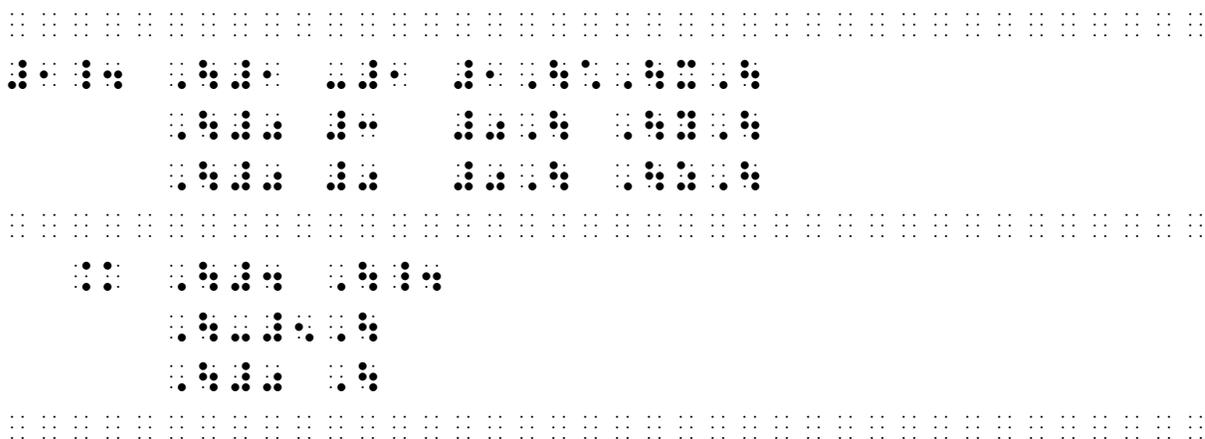


Braille observations: The material outside of the determinant is placed on the top line of the arrangement. The identifier is moved to the left of the displayed expression according to the rules of the Nemeth Code. (See **Lesson 11**, "Displayed Material with Labels".)

Example 16.13-4

$$1. \begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 4 \\ -5 \\ 0 \end{vmatrix} .$$

Print observations: An item number (1.) is centered to the 3×3 array. Also centered within the arrangement are a multiplication dot, an equals sign, and a period.



Braille observations: The material outside of the array is placed on the top line of the arrangement. The multiplication dot is unspaced from the items being multiplied, following spacing rules for operation signs. The entire expression will not fit across the braille page so it is divided before the comparison sign as dictated by the rules of the Nemeth Code.

PRACTICE 16F

1. $\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$

2. $\begin{pmatrix} 1 & -\frac{4}{3} & \frac{5}{3} \\ 2 & 5 & 12 \end{pmatrix}$

3. $\begin{bmatrix} a & b & c \\ 0 & 0 & 0 \end{bmatrix}$

4. $\begin{vmatrix} ab & cd \\ ac & ce \end{vmatrix}$

5. Explain why points (a_1, b_1) , (a_2, b_2) , and (a_3, b_3) are collinear if and only if

$$\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} = 0$$

6. The unit vectors in the direction of the x , y , and z axes of a three dimensional Cartesian coordinate system are

$$\hat{\mathbf{i}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \hat{\mathbf{j}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \text{ and } \hat{\mathbf{k}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Further Considerations with Determinants and Matrices

In order to save space, code switch indicators are omitted from further examples unless both UEB and Nemeth Code are in use. Assume there is a blank line before and after each arrangement.

16.14 Multiplying Arrays: Recall that a multiplication problem can be printed without a multiplication symbol. When each factor is enclosed in grouping signs, it is understood that the side-by-side factors are to be multiplied. For example, $3 \cdot 2$ and $(3)(2)$ both mean "three times two". Similarly, when arrays are being multiplied, the multiplication symbol is often not printed. This array

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

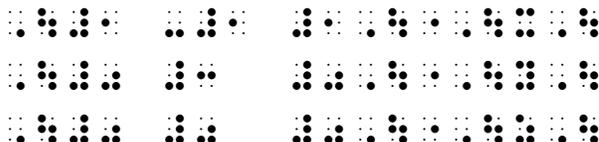
can also be printed without the multiplication dot, like this:

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

The spacing between the arrays shown (in print) is there only to distinguish between the two factors. The space is not brailled. When vertical bars are used, a multipurpose indicator is brailled between the two symbols, otherwise it is misread as a double vertical bar.

Example 16.14-1

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$



The multipurpose indicator is brailled between each side-by-side enlarged vertical bar, on every line of the arrangement.

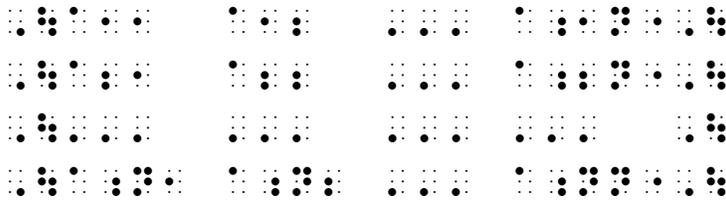
16.15 Omission Dots: When dots are printed in an array to indicate omission of one or more rows or columns, a series of dot 3s is used in braille according to the following observations.

16.15.1 No Dots Are Printed Between Columns: When at least one dot is printed in each column and no dots are printed between columns, each omission is brailled as a 3-cell Nemeth Code ellipsis beginning in the leftmost cell of its column.

Example 16.15-1

$$\begin{vmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \dots & \mathbf{a}_{1n} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \dots & \mathbf{a}_{2n} \\ \cdot & \cdot & \dots & \cdot \\ \mathbf{a}_{n1} & \mathbf{a}_{n2} & \dots & \mathbf{a}_{nn} \end{vmatrix}$$

Print observations: Omissions reading down some columns are printed as one dot; omissions reading across some rows are printed as three dots.



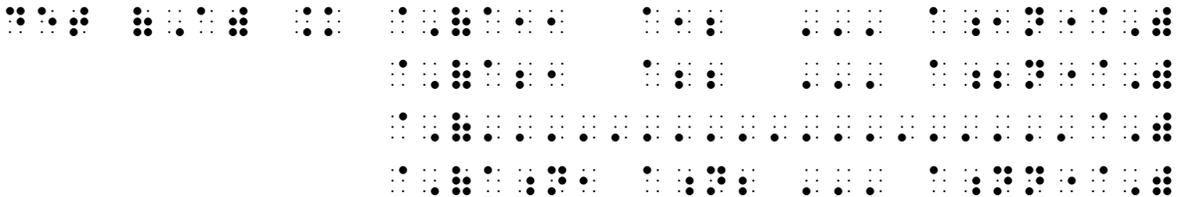
Braille observation: A baseline indicator is needed to assure the closing grouping signs on lines 1, 2, and 4 are on the same level as the opening grouping signs.

16.15.2 Dots Are Printed Between Columns or Some Entries are Blank: Here is the same problem, printed in another style where dots are strung completely across the omitted row (row 3). Notice that the dots occupy space between the columns. A row of omission dots is brailled across the full width of the array.

Example 16.15-2

$$\det(A) = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

Print observation: The third row is printed as a series of dots across the width of the array.



Braille observations: The row of dots is brailled as a series of unspaced dot 3s, beginning in the first cell of the first column and extending to the end of the longest entry in the last column. On lines 1, 2, and 4 a baseline indicator is needed to assure each closing grouping sign is on the same level as the opening grouping sign.

Another print style shows the missing items in the outer columns as single dots, and the missing items in columns 2 and 3 as blank space.

Example 16.15-3

$$\det(A) = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{vmatrix}$$

CHALLENGE PROBLEM #2

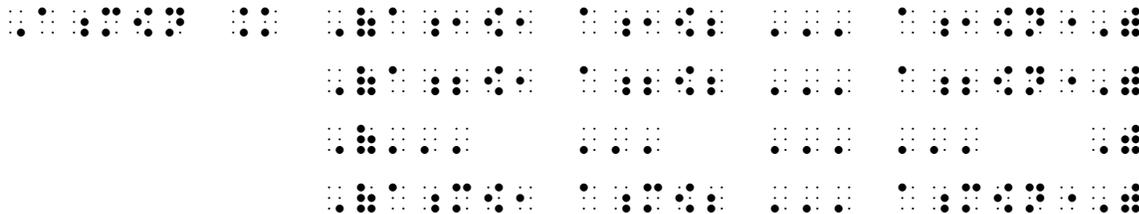
Omissions in an array may also be printed as depicted below.

$$A_{m,n} = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{pmatrix}$$

Print observations: Omissions reading down some columns are printed as three stacked dots (a "vertical" ellipsis); omissions reading across some rows are printed as three dots; one blank entry is printed as an oblique ellipsis.

Which braille rule applies?

As long as at least one dot is printed in each column and no dots are printed between columns, each omission is brailled as a 3-cell Nemeth Code ellipsis beginning in the leftmost cell of its column.



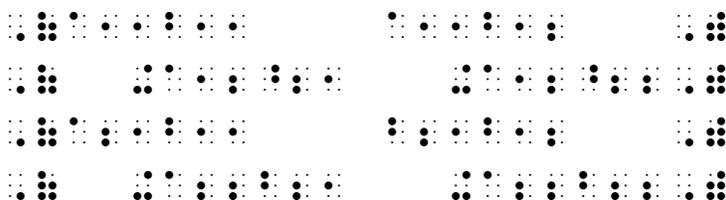
Review rules regarding the contracted comma in Lesson 8.

16.16 Space-Saving Techniques: Sometimes space-saving techniques are required in order to confine the arrangement to one braille page.

16.16.1 Runovers With Indentation: Entries may be run over to a new line. Each runover is indented two cells from the first cell of the first line of the entry. Attempt to apply the priority list regarding division of long expressions, but those rules can be disregarded if there is no other way around the problem. There is no need to explain the runover format to the reader.

Example 16.16-1

$$\begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & b_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

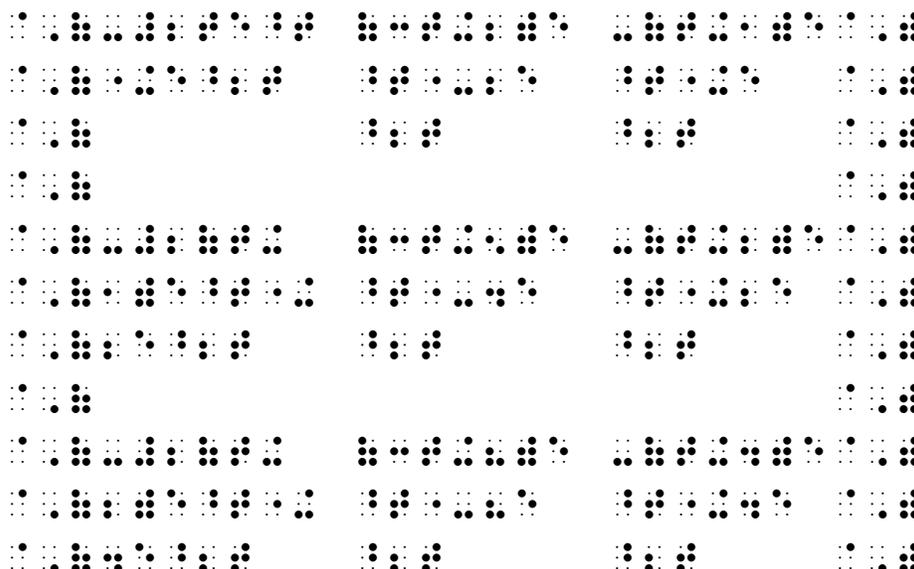


This "runovers with indentation" method is the preferred space-saving technique.

16.16.2 Runovers Without Indentation: If the technique above is not feasible, entries may be run over to new lines without indentation. Preference rules for runovers of mathematical expressions need not be observed if space would be saved. In order to distinguish each row, a blank line is inserted between them. Enlarged grouping symbols are brailled on those blank lines within the arrangement.

Example 16.16-2

$$\left[\begin{array}{ccc} -2te^t + e^{2t} & (3t+2)e^t - 2e^{2t} & -(t+1)e^t + e^{2t} \\ -2(t+1)e^t + 2e^{2t} & (3t+5)e^t - 4e^{2t} & -(t+2)e^t + 2e^{2t} \\ -2(t+2)e^t + 4e^{2t} & (3t+8)e^t - 8e^{2t} & -(t+4)e^t + 4e^{2t} \end{array} \right]$$



16.16.3 Fractions in Arrays: Fractions may be shown spatially if linear fractions take up too much space. Observe the following details.

- A line is skipped between the rows containing the spatial fraction. Enlarged grouping symbols are brailled on those blank lines within the arrangement.
- Operation signs and variables are placed on the same line as the spatial fraction line.
- Each entry is moved as far up as possible in its row. This includes entries that are not fractions.
- If the last item in a row is a superscript, or a subscript that uses a subscript indicator, a return to the baseline must occur before the closing grouping symbol is brailled. If the row extends fully to the closing grouping symbol, a baseline indicator is required to return to the baseline. If the row does not extend to the closing grouping symbol, the space returns the reader to the baseline.

Example 16.16-3

$$\begin{vmatrix} \frac{3}{8}e^{2x} & -\frac{3}{8} & +\frac{1}{4} & -\frac{3}{4}x^2 \\ -\frac{3}{8}e^{2x} & +\frac{3}{8} & +\frac{3}{4} & +\frac{3}{4}x^2 \\ 3e^{2x} & -\frac{3}{8} & -\frac{3}{4} & +\frac{3}{4}x^2 \end{vmatrix}$$

The single-line entry in row 3, column 1, is brailled on the first line of the row even though it is centered in its row in print.

16.16.4 Keying: When the array will not fit on the page even after trying the methods described above, a key may be devised. The technique of keying is discussed in the next lesson.

PRACTICE 16I

1) If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}$, find AB .

Answer: $AB = \begin{bmatrix} 1 & -25 & 10 \\ 29 & 1 & -18 \end{bmatrix}$.

2) Here are three examples of matrix operations.

a) $\begin{bmatrix} 5 & 4 \end{bmatrix} + \begin{bmatrix} 20 & 30 \end{bmatrix} = \begin{bmatrix} 25 & 34 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 1 & 5 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & 7 \\ 1 & 9 \end{bmatrix}$

c) $\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 2 \end{bmatrix}$

16.19 Use of Tactile Graphics for Enlarged or Horizontal Grouping Signs: Enlarged grouping symbols may be drawn in place of the braille equivalents, especially when space saving is a factor or when a horizontal grouping sign is required. The horizontal brace shown below is best presented as a tactile graphic. Refer to BANA's *Guidelines and Standards for Tactile Graphics* for drawing techniques.

$$\begin{aligned} (2x + 3)(x - 5) &= 2x^2 - 10x + 3x - 15 \\ &= 2x^2 \quad \underbrace{-10x + 3x}_{-7x} \quad - 15 \end{aligned}$$

For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL

The margin for material displayed to itemized text is cell 5, even if the itemized text has no runovers.

ANSWERS TO PRACTICE MATERIAL

EXERCISE 16

Exercise 16 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 17.