LESSON 15

Read about this PROVISIONAL EDITION in the front matter to this book. Check the NFB website periodically for updates to this lesson.

- **SPATIAL ARRANGEMENTS OF FRACTIONS**
- **HYPERCOMPLEX FRACTIONS**
- **CONTINUED FRACTIONS**

**SPATIAL ARRANGEMENTS OF FRACTIONS**

Fractions are usually transcribed linearly, as discussed in Lesson 9. However, certain situations require a spatial arrangement. Before addressing those conditions, the construction of a spatially-arranged fraction will be illustrated. Recall the terminology used for the parts of a fraction.

\[
\begin{array}{c}
& \text{numerator} \\
3 & \frac{3}{4} & \text{fraction line} \\
4 & \text{denominator}
\end{array}
\]

**15.1 Spatial Fraction Line:** The spatial fraction line is comprised of a series of dots 25. An opening fraction indicator (1456) marks the beginning of the fraction line; a closing fraction indicator (3456) signals the end.

In a spatially-arranged fraction the numerator is placed above the spatial fraction line and the denominator is placed below it. The fraction line is comprised of the same number of cells as the longest line of the braille above (numerator) or below (denominator), with the fraction indicators placed one cell beyond the width on either end, as illustrated in the next section.

**15.2 Numerator and Denominator:** Since fractions do not contain material aligned for computation, the numeric indicator must be used in the numerator and/or denominator according to the rules of the Nemeth Code.

\[
\begin{array}{c}
\text{Example 15.2-1} & \frac{3}{4} \\
\end{array}
\]

Likewise, the English letter indicator must be used where necessary in the numerator and/or denominator.

\[
\begin{array}{c}
\text{Example 15.2-2} & \frac{x}{y} \\
\end{array}
\]
Terms of the fraction are centered above and below their fraction lines.

**Example 15.2-3**

\[
\frac{3}{100}
\]

If exact centering is not possible, the term is moved to the left one cell.

**Example 15.2-4**

\[
\frac{25}{100}
\]

**15.3 Placement of Identifiers with Spatially Arranged Fractions:** An identifier, if present, is placed on the same braille line as the fraction line. One blank space is left between the last symbol in the identifier and the symbol furthest left in the overall arrangement.

**Example 15.3-1**

\[
\frac{cd}{ef}
\]

**Situations Requiring Spatial Presentation**

**15.4 Simple Fractions Arranged Spatially for Illustration:** When the parts of a simple fraction are labeled, it may be helpful to use a spatial arrangement. Labels are included within the code switch and are uncontracted.

**Example 15.4-1**

Four children share a dozen cookies equally. Expressed as a fraction, how many cookies does each child receive?

\[
\frac{12}{4}
\]
PRACTICE 15A

1. Shandra invited seven friends to a pizza party. Two pizzas were ordered. Each pizza had eight slices. Which fraction shows how many slices of pizza each child can have if they share equally?

   a. \( \frac{2}{8} \) number of pizzas
      \( \frac{8}{8} \) number of children
   
   b. \( \frac{8}{8} \) number of slices
      \( \frac{8}{8} \) number of children
   
   c. \( \frac{16}{8} \) number of slices
      \( \frac{8}{8} \) number of children

15.5 Cancellation Within Fractions: Recall that a spatial arrangement is required whenever numbers, letters, or abbreviations are canceled in print by any type of stroke through them.

<table>
<thead>
<tr>
<th>Opening Cancellation Indicator</th>
<th>☯ ☯ ☯</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closing Cancellation Indicator</td>
<td>☯ ☯ ☯</td>
</tr>
</tbody>
</table>

When cancellation occurs within a fraction, replacement items (the result of cancellation) are placed above canceled items in the numerator and below canceled items in the denominator. Each replacement item is centered with respect to the canceled term.

Example 15.5-1

\[ \frac{1 \text{ replacement item centered above canceled } 25}{\frac{25}{50}} \]

Line 1: Replacement "1" is centered above canceled "25". A numeric indicator is required.
Line 2: A numeric indicator is not required because the numeral is not preceded by a space.
Line 3: The fraction line is as long as the longest line in the fraction – in this case, both numerator and denominator have a 4-cell item.
Line 4: A numeric indicator is not required because the numeral is not preceded by a space.
Line 5: Replacement "2" is centered below canceled "50". A numeric indicator is required.
Fractions which do not require a spatial arrangement are brailled using the linear method.

Example 15.5-2

\[
\frac{25}{50} = \frac{1}{2}
\]

Line 3: The second fraction ("one-half") is brailled as a linear fraction because it does not contain cancellation. The equals sign and the linear fraction are brailled on the same line as the spatial fraction line.

If exact centering is not possible, the term is moved to the left one cell.

Example 15.5-3

\[
\frac{1}{6} = \frac{1}{6}
\]

Line 1: Replacement "1" is centered above canceled "6". Exact centering is not possible so it is moved left one cell.
Line 2: Canceled "6" is centered above the fraction line. Exact centering is not possible so it is moved left one cell.
Line 3: The fraction line is as long as the longest line in the fraction (the denominator).
Line 4: Canceled "36" is centered below the fraction line.
Line 5: Replacement "6" is centered below canceled "36".
More than one level of cancellation may be encountered. *Reminder: The print copy may show any type of stroke to indicate cancellation.*

**Example 15.5-4**

\[
\begin{array}{c}
1. \frac{22}{88} \\
18 \times 5280 \\
60 \times 60 \\
\hline \\
1 \\
\end{array}
\]

*Reminder: Vertical alignment of factors is not a consideration in spatial fractions because the material is not being aligned for computation. Instead, terms of the fraction are centered above or below their fraction lines, and replacement items are centered above or below their canceled item.*

**Clarification:** Placement of the replacement item is relative to the width of the canceled item, without respect to place value.

\[
\begin{array}{c}
3 \\
1449 \\
\hline
\end{array}
\]

**15.5.1 Extent of Cancellation:** Items which are individually canceled in print are individually canceled in the transcription. Compare the next two examples.

**Example 15.5-5**

\[
\begin{array}{c}
16x \\
16y \\
\hline
\end{array}
\]

*Each letter "r" "s" and "t" is printed with a slash through it. Letter indicators are not required because each letter is touching an indicator.*

**Example 15.5-6**

\[
\begin{array}{c}
(rst) \\
(rst)y \\
\hline
\end{array}
\]

*Each letter combination "rst" is printed with a slash through it.*
15.5.2 Cancellation and Level Indicators: Paired cancellation indicators must be placed on the same level of writing.

"d^2" is canceled. The cancellation indicators are on the baseline of writing.

Example 15.5-7
\[
\frac{d^2}{1} = d
\]

The replacement items are centered above and below the canceled items.

In the next example, only the superscript is canceled.

The cancellation indicators are in the superscript position.

Example 15.5-8
\[
\frac{4x^2q}{p^2} = \frac{4x^2q}{p^2}
\]

The replacement numeral "2" is placed below the canceled superscript in the denominator, centered below the term it replaces.

15.5.3 Canceled Abbreviations: A canceled abbreviation is unspaced from the cancellation indicators.

The spaces required before and after the abbreviation are inserted before the opening cancellation indicator and after the closing cancellation indicator.
15.6 Chemistry Exception: An exception is made in the Chemistry Code regarding cancellation in chemical equations. Refer to Braille Code for Chemical Notation 1997 for details.

---

PRACTICE 15B

1. How many meters are in 100 yards? By multiplying 100 yd by several expressions of "1" and canceling units of measure, an answer is found: There are 191.44 m in 100 yd.

\[ ? \text{ m} = 100 \text{ yd} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{1 \text{ m}}{100 \text{ cm}} \]

All lines: A space is required between each abbreviation and its related numeral (line 1) even when the abbreviation is canceled (lines 2 and 3).

Line 2: The abbreviation "in." is spaced away from the operation symbol (multiplication dot) even when enclosed between cancellation indicators.

\[ = \frac{100 \times 3 \times 12 \times 2.54}{100} \text{ m} \]

\[ = 91.44 \text{ m} \]

2. \[ \frac{8 \times 8}{1} = \frac{1 \times 8}{1} = 8 \]

3. \[ \frac{2b+a}{b} \div \frac{4b^2-a^2}{b^2} = \frac{1}{\frac{2b+a}{b}} \cdot \frac{b}{\frac{(2b+a)(2b-a)}{1}} = \frac{b}{2b-a} \]

4. Simplify using cross-canceling. Multiply the numerators and multiply the denominators.

\[ \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \]
HYPERCOMPLEX FRACTIONS

15.7 Definition and Recognition: A hypercomplex fraction is one whose numerator or denominator, or both, contain at least one complex fraction. (The term "hypercomplex" fraction is used only in the context of the Nemeth Code—it is unlikely that you will encounter this term in a math book.)

Recall from Lesson 9 that a complex fraction is one whose numerator and/or denominator are, or contain, one or more simple fractions or mixed numbers. In other words, a complex fraction is a fraction within a fraction. Here is an example.

Complex Fraction

\[
\frac{\frac{1}{2}}{5}
\]

Here is a print example of a hypercomplex fraction where the denominator is "5" and the numerator contains the complex fraction "one-half over three-fourths."

\[
\frac{\frac{1}{2}}{\frac{3}{4}}
\]

This is a hypercomplex fraction.

A fraction is not a hypercomplex fraction if the only complex fractions it contains are at the superscript or subscript level. The following is not a hypercomplex fraction. The numerator is "1" and the denominator is "2 raised to the (one-half over three-fourths) power."

\[
\frac{1}{\frac{1}{2}^{\frac{3}{4}}}
\]

This is not a hypercomplex fraction.

This fraction is brailled linearly, as a simple fraction whose denominator contains a complex fraction in the superscript position. Review the transcription of complex fractions in Lesson 9.

15.8 Hypercomplex Fraction Indicators: Opening and closing hypercomplex fraction indicators mark the beginning and end of a hypercomplex fraction or the beginning and end of a hypercomplex fraction line in a spatial arrangement.

<table>
<thead>
<tr>
<th>Hypercomplex Fraction Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Spatial Hypercomplex Fraction Line with opening and closing fraction indicators

(varying length)
Here is a transcription of the example from the previous page where the complex fraction in the numerator is brailled as a linear arrangement within the overall spatial arrangement.

\[
\frac{\frac{\frac{1}{2}}{3}}{4} \frac{5}{22/2019 \text{ revision}}
\]

Example 15.8-1

\[
\frac{\frac{1}{2}}{\frac{3}{4}} \frac{5}{22/2019 \text{ revision}}
\]

Terms of the fraction are centered above and below their fraction lines.

If the numerator and denominator are too long to fit on one line, follow the rules regarding division of mathematical expressions between braille lines. (See Lesson 14.)

Example 15.8-2

1. Show that \(\frac{\frac{3\frac{1}{7}}{7\frac{1}{5}} - \frac{1}{2}}{\frac{2\frac{7}{7}}{4\frac{1}{5}} \times \frac{2\frac{2}{3}}{6\frac{3}{8}}}\) of \(\frac{11\frac{1}{5}}{17} \times 52 \frac{4}{11} = \frac{1}{2}\).

Numerator and denominator are divided before an operation sign (priority #2). The numerator is divided before the minus sign; the denominator is divided before the multiplication cross. Terms of the fraction are centered above and below the spatial fraction line.
The combined transcription method illustrated above is the preferred way of brailling a hypercomplex fraction. However, it is permissible to use an entirely spatial arrangement or an entirely linear arrangement. Example 15.8-1 is illustrated below using these two alternate methods.

\[
\begin{array}{c}
\frac{1}{2} \\
\frac{3}{4} \\
5
\end{array}
\]

Entirely spatial

Entirely linear

A linear arrangement uses the horizontal hypercomplex fraction line.

Linear hypercomplex fractions are difficult to assemble mentally.

15.9 Higher Orders of Complexity: Hypercomplex fractions of higher order may be transcribed in the manner described above, with dot 6 added the appropriate number of times before the fraction indicators and the matching fraction lines. No examples are shown.

Instructions: Transcribe this hypercomplex fraction using the combined method—that is, braille each complex fraction as a linear arrangement within the overall spatial arrangement. Numerators and denominators can be determined by noting the length of each fraction line in print.

**PRACTICE 15C**

\[
\begin{array}{c}
\frac{r^2 - 4s^2}{s^2} \\
\frac{s^2}{r + 2s} \\
s \\
\frac{4r - 2s^2}{3s} \\
\frac{2s^2 - 3r}{4r}
\end{array}
\]
CHALLENGE PROBLEM

This transcription uses the combined method of brailling a hypercomplex fraction. The numerator is "1". The complex fraction in the denominator is brailled as a linear arrangement within the overall spatial arrangement. There are runovers in the denominator. The runover sites are selected according to the priority list (see Lesson 14). Runovers are centered to the fraction line.

Assuming this complex fraction is displayed to a narrative paragraph, the leftmost braille cell of the fraction will be placed in cell 3.

\[
\frac{1}{\frac{mn}{m+n}(\bar{x}_1 - \bar{x}_2)^2} = \frac{\sum(x_1 - \bar{x}_1)^2 + \sum(x_2 - \bar{x}_2)^2}{\sum(x_1 - \bar{x}_1)^2 + \sum(x_2 - \bar{x}_2)^2}
\]
CONTINUED FRACTIONS

15.10 Definition and Recognition: A continued fraction is one in which each denominator is the sum of a whole number and a fraction. Such a fraction is transcribed entirely as a spatial arrangement.

\[
\frac{n}{\text{Definition} \quad \text{and Recognition}: \quad \text{A continued fraction is one in which each denominator is the sum of a whole number and a fraction. Such a fraction is transcribed entirely as a spatial arrangement.}}
\]

\[
\frac{n}{\text{Example 15.10-1} \quad n = 1 - \frac{7}{3 + \frac{3}{2 + \frac{2}{2 - \frac{2}{3}}}}}
\]

Description: The fraction begins with \( 1 - \frac{7}{3} \) ... the first denominator is \( 3 + \frac{3}{2} \) ... the next denominator is \( 2 + \frac{2}{2} \) ... the final denominator is \( 2 - \frac{2}{3} \).

Opening and closing fraction indicators are not used within a continued fraction. Each fraction line (dots 25) is proportionately the same length shown in print. Each numerator is centered with respect to the fraction line below it. If the numerator cannot be exactly centered, place it one cell to the left of center.

1-2: The opening Nemeth Code indicator is brailled on the line preceding the required blank line.

Line 3: The numerator is centered over its spatial fraction line.

Line 4: The length of this fraction line extends to the rightmost character in the arrangement.

Line 5: This numerator "3" is centered over the spatial fraction line below it. Likewise for the numerators on lines 7 and 9.

Line 6: The first character (numeric indicator) is directly aligned with the first cell of the fraction line two lines above. Likewise for lines 8 and 10.

Line 11: This final denominator consists of two cells. The fraction line above it is the same length.

Lines 12-13: The Nemeth Code terminator is in cell 1 on the line following the required blank line.
PRACTICE 15D

\[ \sqrt{3} = 1 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \ldots}}}. \]

**FORMAT**

**Remarks and Comments**

15.11 Guidelines: When short narrative comments alternate with math problems, switch out of Nemeth Code to transcribe the comments in contracted braille. Two ways to handle such remarks are illustrated in this section. Regardless of which method you use, the same format should be used throughout the transcription and an explanation should be included on the Transcriber's Notes page according to the guidelines given in *Braille Formats*.

Author's comments are often printed in a different color or typeface. The distinction is disregarded in the braille transcription according to guidelines given in *The Rules of Unified English Braille*. In the examples in this section, author's remarks are printed in italics.

15.11.1 OPTION #1—Continue the Commentary on the Same Line: This option is the preferred treatment of authors' remarks. The comments continue on the line on which the related math ends, with runovers in the appropriate location for the math expression. By placing the opening Nemeth Code indicator after the last word of each comment rather than at the beginning of the next line of math, the switch indicator does not interfere with the math under study. If there is not room on the line, the indicator is treated as a runover to the comment line.
Example 15.11-1

If \( y \) varies inversely as \( x \), and \( y = 3 \) when \( x = 4 \), find \( y \) when \( x = 18 \).

\[
\frac{x_1}{y_2} = \frac{x_2}{y_1}
\]

\[
\frac{4}{y_2} = \frac{18}{3}
\]

*Substitute the known values.*

\[
18y_2 = 12
\]

*Now cross multiply.*

\[
y_2 = \frac{12}{18} \quad \text{or} \quad \frac{2}{3}
\]

*Divide each side by 18 and simplify.*

The value of \( y \) when \( x = 18 \) is \( \frac{2}{3} \).

---

Lines 1-3: Narrative begins in cell 3, runovers in cell 1.
Lines 4, 5, 7, 9: Each displayed math expression begins in cell 3.
Lines 5, 7, 9: A Nemeth Code terminator switches the reader to UEB for the comments, which follow on the same line as the math expression. The italic typeform for the comments has no meaning and is not retained.
Lines 6, 8, 10: Comment runovers begin in the standard position for displayed material. In this example, that is cell 5.
Lines 6 and 8: The opening Nemeth Code indicator is placed at the end of the line of text preceding the technical material so that all of the displayed math expressions may begin in the same cell.

*Note that, on lines 2-3, "find \( y \) when \( x = 18 \)" can also be transcribed as follows:*
The single-word switch indicator is used for a one-word comment.

Example 15.11-2

What percent of 48 is 54?

\[ n \cdot 48 = 54 \quad \text{Write an equation.} \]

\[ \frac{48n}{48} = \frac{54}{48} \quad \text{Divide each side by 48.} \]

\[ n = 1.125 \quad \text{Simplify.} \]

\[ = 112.5\% \quad \text{Change the decimal to a percent.} \]

54 is 112.5% of 48.
15.11.2 OPTION #2—Indent the Commentary on the Next Line: This layout provides clarity when a comment is also a math expression or when a comment refers to a spatial arrangement. Begin the remark on the line following the related math expression, indented four cells to the right of the current runover cell. Runovers to the commentary are blocked in the same cell. Whether switching into or out of Nemeth Code, place the switch indicators after the last item of the line rather than at the beginning of the next line. This maintains clarity in the indented margin pattern. Remember, if this method is used in one instance, this format must be followed throughout the transcription.

Example 15.11-3

1. Reduce \( \frac{330}{4950} \) to lowest terms using the "prime factors" method.

\[
\frac{330}{4950} = \frac{2 \times 3 \times 5 \times 11}{2 \times 3 \times 5 \times 5 \times 11}
\]

Express numerator and denominator as prime factors.

\[
= \frac{2 \times 3 \times 5 \times N}{2 \times 3 \times 5 \times 5 \times N}
\]

Cancel common factors 2, 3, 5, and 11.

\[
= \frac{1}{15}
\]

3 \times 5 (or 15) remains in the denominator. (1 is implied in the numerator.)
Line 3: Math displayed to itemized (1-3) begins in cell 5.
Lines 3-17: This linked expression meets the criteria for "special margins". The anchor starts in cell 5 (line 3). Each link begins in cell 7 (lines 4, 11, 17). Runovers begin in cell 9 (line 5).
Lines 6, 15, 18: Each comment begins in cell 13, which is four cells to the right of the runover position of the previous line, even if no runover is present.
Lines 7, 16, 19-20: Runovers to comments are blocked in cell 13.
Line 16: The switch indicator is brailled at the end of the comment in order not to interfere with the alignment of the equals sign that begins the next link on line 17.

Instructions: Demonstrate your understanding of the rules by transcribing this practice twice – first using option #1, then using option #2. NOTE: In print, the equals signs are aligned. This alignment is disregarded in the braille transcription.

**PRACTICE 15E**

We write the equation in the slope-intercept form, $y = mx + b$, by solving for $y$.

$$-5x - 2y = 6$$

$$-2y = 5x + 6 \quad Add \ 5x \ to \ both \ sides.$$  

$$\frac{-2y}{-2} = \frac{5x+6}{-2} \quad Divide \ both \ sides \ by \ -2.$$  

$$y = \frac{5x}{-2} + \frac{6}{-2} \quad Divide \ each \ term \ by \ -2 \ and \ simplify.$$  

$$y = -\frac{5}{2}x - 3 \quad Slope-intercept \ form$$
15.12 Definition and Recognition: A stem-and-leaf plot is a method of showing data distribution in columns and rows. A vertical line partitions the information into data on the left, called the stem, and data on the right, called the leaf. Stem and leaf data may consist of numbers, letters, and/or blank entries and may be spaced or unspaced in the print copy. Stem data are right justified to the vertical line; leaf data are left justified to the vertical line. The columns may include a heading. Here is an example.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0 4</td>
</tr>
<tr>
<td>9</td>
<td>3 4 5 7 8 9</td>
</tr>
<tr>
<td>8</td>
<td>0 2 2 9</td>
</tr>
<tr>
<td>7</td>
<td>5 8 9</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

A distinctive feature of a stem-and-leaf plot is the vertical line separating the stems from the leaves.

15.13 The Table: A stem-and-leaf plot is brailled in Nemeth notation using the guidelines for tables outlined in *Braille Formats*. (An introduction to table format in technical materials was given in Lesson 6.) The entire table is transcribed in Nemeth Code, including the column headings (if present). Column headings begin in cell 1 and no contractions are used. Column separation lines follow on the next line below the headings, covering the same number of cells as the widest data entry in each column. Column headings are not brailled if they do not appear in print.

The vertical line between the stem column and the leaf column is brailled as dots 456, preceded and followed by one blank cell. This line is brailled in every row, including between column headings and between column separation lines. The presence of this Nemeth Code symbol necessitates the use of Nemeth Code throughout the table.

| Vertical Line (spaced) | :: :: |
The following Nemeth Code rules apply to this example.

— The numeric indicator is omitted in the body of the plot.
— Spaces are not brailled between single-digit entries.
— The data are justified to the vertical line as shown in print.

Example 15.13-1

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0 4</td>
</tr>
<tr>
<td>9</td>
<td>3 4 5 7 8 9</td>
</tr>
<tr>
<td>8</td>
<td>0 2 2 9</td>
</tr>
<tr>
<td>7</td>
<td>5 8 9</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

The table is preceded and followed by a blank line. Code switch indicators do not replace required blank lines.
15.14 The Key: A key is often provided in the print copy and may be located above, below, next to, or within the table. The key is brailled first, regardless of its placement in the print copy. Numeric indicators are not used for the portion of the key that replicates a numeric entry in the plot. (In this illustration, 10|0 is the portion of the key replicating the stem "10" and the leaf "0".) The value assigned to the key is brailled using the numeric indicator as required by the Nemeth Code. (In this illustration, a numeric indicator is used for the value "100").

\[ 10 \mid 0 = 100 \]

The numbers in this key are printed in italics. Typeform is not retained in braille. A key is not constructed by the transcriber if one does not appear in print.

The following Nemeth Code rules apply to this example.

— The key is brailled first, beginning in cell 1.
— The numeric indicator is omitted in the body of the plot.
— Spaces are not brailled between single-digit entries.

Example 15.14-1

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0 4</td>
</tr>
<tr>
<td>9</td>
<td>3 4 5 7 8 9</td>
</tr>
<tr>
<td>8</td>
<td>0 2 2 9</td>
</tr>
<tr>
<td>7</td>
<td>5 8 9</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

\[ 10 \mid 0 = 100 \]
If the key requires a runover line, the runover begins in cell 3. When there are words in the key it may be brailled in UEB, switching to Nemeth Code for the portion that reflects the stem and leaf. This key also includes a descriptive label ("Key:"). A label is brailled only if it appears in print. If the key is set off by shading or in a box, box lines are not brailled.

**Example 15.14-2**

<table>
<thead>
<tr>
<th>Key: 2</th>
<th>2 represents 2,150,000 to 2,249,999 copies sold.</th>
</tr>
</thead>
</table>

**Example 15.15 Data Consisting of More Than One Character; Punctuation Between Entries:** The next example shows leaf entries with a decimal point and more than one digit. It also shows commas between leaf entries. The key is printed within the body of the table. The following rules are applied:

- The key is brailled first.
- The numeric indicator is omitted in the body of the plot.
- Data consisting of groups of two or more digits require one blank cell between entries.
- Punctuation printed between units of data is omitted in the transcription.

**Example 15.15-1**

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>8.3</td>
</tr>
<tr>
<td>6</td>
<td>4.3, 5.1, 5.5, 6.7, 7.0, 8.7, 9.3</td>
</tr>
<tr>
<td>7</td>
<td>0.0, 2.8, 3.2, 5.8, 7.4, 7.4</td>
</tr>
</tbody>
</table>

| Key: 5 | 8.3 represents 5,800,000 to 5,839,999 copies sold. |
15.16 Alphabetic Data: For alphabetic data, the following rules apply.

— The English letter indicator is omitted in the body of the plot.
— Spaces are not brailled between single-letter data entries.
— Data consisting of groups of two or more letters require one blank cell between entries.

The leaves in the next example consist of pairs of lowercase letters. There is no key.

Example 15.16-1

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>aa ab ac ba bd cc cd ef en er</td>
</tr>
<tr>
<td>42</td>
<td>bd bd dc</td>
</tr>
</tbody>
</table>

Reminder: The numeric indicator is omitted in the body of the plot (Stem column).

15.16.1 Alphabetic Key: In an alphabetic key, the English letter indicator is not used for the portion of the key that replicates an entry in the plot. The value assigned to the key is brailled using the English letter indicator as required by the Nemeth Code. If the key is printed in a box, box lines are not brailled.

\[ c | d = cd \]

Reminder: "cd" does not require an English letter indicator because it immediately follows a sign of comparison. See 4.16.2 in Lesson 4.
15.17 Blank Entries: A blank entry in a stem-and-leaf plot is shown as blank space in the braille transcription. Do not braille a general omission symbol. Do not fill the width of the column with dot 5s.

The following rules also apply to the next example.

— The numeric indicator and the English letter indicator are omitted in the body of the plot.
— Spaces are not brailled between single-letter data entries.
— Uppercase letters are capitalized individually.

Example 15.17-1

<table>
<thead>
<tr>
<th></th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td></td>
</tr>
<tr>
<td>96</td>
<td>M</td>
</tr>
<tr>
<td>97</td>
<td>M</td>
</tr>
<tr>
<td>98</td>
<td>M</td>
</tr>
<tr>
<td>99</td>
<td>M M A M</td>
</tr>
</tbody>
</table>

This plot has no key and no column headings—none are added in braille. The blank space in the second row indicates a blank entry in the leaf column. Single letters are unspaced and are capitalized individually.
15.18 Runovers Within the Table: A runover of leaves shown in print should be ignored if all leaves will fit on one row in braille. Use the full width of the available cells in the row.

Example 15.18-1

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 2 2 3 4 4 5 5 5 5 6 6 7</td>
</tr>
<tr>
<td></td>
<td>7 7 7 8 8 9 9 9 9 9 9 9 9</td>
</tr>
<tr>
<td>2</td>
<td>0 1 1 1 2 6 6 8</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

The first row of leaf data requires two lines in print but not in braille.
If leaf data will not fit on one line in the braille transcription, indent the runover line two cells to the right after using as much of the braille line as possible. Do not repeat the stem row heading. The next example illustrates, expanding the two-letter data example shown previously.

Example 15.18-2

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>aa   ab ab ac ba bd cc cd cd ef en er er</td>
</tr>
<tr>
<td>42</td>
<td>bd   bd dc</td>
</tr>
</tbody>
</table>

The vertical line is brailled on every line in the table.

15.19 Back-To-Back Plot: A back-to-back stem-and-leaf plot is used when two sets of data are to be compared. There are three columns. The stem is the middle column. Data is read from the stem outward, which means that data in the left leaf is read from right to left. There will be two keys. In this example, the keys are printed within the body of the table.

Example 15.19-1

Ms. Abel's Test Scores

<table>
<thead>
<tr>
<th>Second Grade Classes</th>
<th>Fifth Grade Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>453150</td>
<td>score of 50</td>
</tr>
<tr>
<td>987776655521</td>
<td>7</td>
</tr>
<tr>
<td>999998888776655444332110</td>
<td>8</td>
</tr>
<tr>
<td>98877753320</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>00</td>
</tr>
</tbody>
</table>
Keys: The left column key is brailed first, followed on the next line by the right column key.

Columns: Column separation lines are brailed below only the lines that contain column headings (line 8). In this table, the stem column (middle column) has no heading.

Runovers:
- Right leaf data are read from left to right; runovers to the right leaf data column are two cells to the right (line 12).
- Left leaf data are read from right to left; runovers to the left leaf data column are two cells to the left (line 14).

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms.</td>
<td>Abel's</td>
<td>site</td>
<td>scores</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Second</td>
<td>grade</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>classes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
</tbody>
</table>

Line 3: Key to left column.

Line 4: Key to right column.

Line 16: The blank left-leaf entry in the last row is a blank space in braille.

Code Switch Decision: The opening Nemeth Code indicator needed for the table falls at the end of line 4. An alternate method would be to place it on a new line in cell 1. A blank line would then be necessary both after the key list and before the table. This alternate method is shown on the next page.
**ALTERNATE METHOD**

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7 8</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0 6 8 8 8</td>
</tr>
<tr>
<td>3</td>
<td>0 2 6 6 7 8</td>
</tr>
<tr>
<td>4</td>
<td>1 2 6 6</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

*Key: 2 | 0 represents 20 guinea pigs*

How many stores have fewer than 36 guinea pigs?

---

**PRACTICE 15F**

A pet store owner constructed the following stem-and-leaf plot showing the number of guinea pigs at each of her seventeen stores:

For further practice, see Appendix A—Reading Practice.
ANSWERS TO PRACTICE MATERIAL

PRACTICE #AE

1A. imagine saving for pizza ipy.
   two pizzas + ordering. Each pizza 16
   eight slices. The fraction is 80. LM
   slices ÷ pizza EAE E E E E E E E LM.
   equally: LM
   
   : number of pizzas
   1A: q: q
   : number of children
   1A: q: q
   : number of slices
   1A: q: q
   : number of children
   1A: q: q
   : number of slices
   1A: q: q
   : number of children

   Regarding the last line, recall that, following a spatial arrangement, the Nemeth Code
   terminator is placed in cell 1 by itself on the line following the required blank line.
In #1, the fourth and fifth line may also be transcribed as follows. In either case, the number "100" must be braille on the same line as its abbreviation "yd".
#1 (2B + A) - A^2

\[(2B + A)(2B - A)
\]

\[
\frac{B}{2B - A}
\]

Simplify using cross-multiplication. Multiply numerators and multiply denominators.
OPTION #1

We write an equation: slope = \( \frac{y_2 - y_1}{x_2 - x_1} \) by solving for \( y \):

\[
\begin{align*}
\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\
&\text{Add } x_1 \text{ to both sides: } x_1 \\
&\text{Divide both sides by } x_2 \\
&\text{Simplify: } x_0
\end{align*}
\]

OPTION #2

We write an equation: slope = \( \frac{y_2 - y_1}{x_2 - x_1} \) by solving for \( y \):

\[
\begin{align*}
\text{Slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\
&\text{Add } x_1 \text{ to both sides: } x_1 \\
&\text{Divide both sides by } x_2 \\
&\text{Simplify: } x_0
\end{align*}
\]
A PET TONE ING + STRUCT + FOLL
SEM-WULIF PLOT KY K NUBLE R GUEEA PIGS
THE EAK + HG SENE-TE: MONES

KEY: LN 3 0 00 REPRES 3 GUEEA
PIGS LN

STEM | LEAF
1111 | 1 1 1 1
1111 | 1 1
1 | 1
1 | 1 1 1 1 1
1 | 1 1 1 1 1
1 | 1 1 1 1
1 | 1 1 1 1
1 | 1 1 1
1 | 1 1 1
1 | 1 1

THE LM MORES B FEW GAN WCF GUEEA
PIGS

An alternate method is shown on the next page.
EXERCISE 15

Exercise 15 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 16.