LESSON 14

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *FUNCTION NAMES AND THEIR ABBREVIATIONS*
- *Division of Mathematical Expressions Between Braille Lines*

Spatial Arrangements
  - *SQUARE ROOT DIVISION*
  - *OTHER PRINT LAYOUTS SHOWING DIVISION*

FUNCTION NAMES AND THEIR ABBREVIATIONS

14.1 Functions: The most common function names as well as their abbreviations are listed below. Function names or their abbreviations that do not appear in this list are subject to the same rules taught in this lesson.

ABBREVIATED FUNCTION NAME	BRAILLE EQUIVALENT	UNABBREVIATED FUNCTION NAME
amp	⠁⠍⠏	amplitude
antilog	⠁⠝⠞⠊⠇⠕⠛	antilogarithm
arc	⠁⠗⠉	arc
arg	⠁⠗⠛	argument
colog	⠉⠕⠇⠕⠛	cologarithm
cos	⠉⠕⠎	cosine
cosh	⠉⠕⠎⠓	hyperbolic cosine
cot	⠉⠕⠞	cotangent
coth	⠉⠕⠞⠓	hyperbolic cotangent
covers	⠉⠕⠧⠑⠗⠎	coversine
csc	⠉⠎⠉	cosecant
csch	⠉⠎⠉⠓	hyperbolic cosecant
ctn	⠉⠞⠝	cotangent
ctnh	⠉⠞⠝⠓	hyperbolic cotangent

(continued)

ABBREVIATED FUNCTION NAME	BRAILLE EQUIVALENT	UNABBREVIATED FUNCTION NAME
det	⠙⠑⠞	determinant
erf	⠑⠗⠋	error function
exp	⠑⠭⠏	exponential
exsec	⠑⠭⠎⠑⠉	exsecant
grad	⠛⠗⠁⠙	gradient
hav	⠓⠁⠧	haversine
im	⠊⠍	imaginary part
inf	⠊⠝⠋	infimum
lim	⠇⠊⠍	limit
$\overline{\text{lim}}$ or $\overline{\text{limit}}$	⠣⠇⠊⠍	upper limit
$\underline{\text{lim}}$ or $\underline{\text{limit}}$	⠩⠇⠊⠍	lower limit
ln	⠇⠝	natural logarithm
log	⠇⠕⠛	logarithm
max	⠍⠁⠭	maximum
min	⠍⠊⠝	minimum
mod	⠍⠕⠙	modulo
re	⠗⠑	real part
sec	⠎⠑⠉	secant
sech	⠎⠑⠉⠓	hyperbolic secant
sin	⠎⠊⠝	sine
sinh	⠎⠊⠝⠓	hyperbolic sine
sup	⠎⠥⠏	supremum
tan	⠞⠁⠝	tangent
tanh	⠞⠁⠝⠓	hyperbolic tangent
vers	⠧⠑⠗⠎	versine

14.1.1 Code Switching: Abbreviated function names are mathematical expressions and are transcribed following the rules of the Nemeth Code – they are not considered to be ordinary abbreviations.

Example 14.1-1 "log" is an abbreviated function name.

⠸⠩ ⠦⠇⠕⠛⠸⠴ ⠸⠱ ⠊⠎ ⠁⠝ ⠁⠆⠗⠑⠧⠊⠁⠞⠫ ⠋⠥⠝⠉⠰⠝
⠐⠝⠲

A punctuation indicator is required before the closing quotation mark in a mathematical expression.

Unabbreviated function names which are not associated with a math term are transcribed in UEB and appropriate contractions are used.

Example 14.1-2 "log" is the abbreviated function name for "logarithm".

⠸⠩ ⠦⠇⠕⠛⠸⠴ ⠸⠱ ⠊⠎ ⠮ ⠁⠆⠗⠑⠧⠊⠁⠞⠫ ⠋⠥⠝⠉⠰⠝
⠐⠝ ⠿ ⠦⠇⠕⠛⠜⠊⠹⠍⠴⠲

When an unabbreviated function name is used in conjunction with an abbreviated function name, a switch to Nemeth Code is required.

Example 14.1-3 What is the meaning of logsine?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠍⠂⠝⠬ ⠷ ⠸⠩ ⠇⠕⠛⠎⠊⠝⠑ ⠸⠱⠦

"log" is an abbreviated function name, so "logsine" requires a switch to Nemeth Code.

For the remainder of this lesson, the term "function name" refers to both abbreviated and unabbreviated function names. If a distinction is necessary, the specific term will be used.

14.2 Spacing of Function Names: Within a mathematical expression, the following spacing rules are observed: no space is to come before a function name; a space is required after a function name. These spacing rules apply regardless of the spacing used in the print copy.

⫸ 3 cos 20° ⠼⠒⠉⠕⠎ ⠼⠆⠴⠘⠨⠡

⫸ Sine 195° ⠠⠎⠊⠝⠑ ⠼⠂⠔⠢⠘⠨⠡

⫸ sin θ ⠎⠊⠝ ⠨⠹

Example 14.2-1 "sin θ" is pronounced "sine theta".

⠸⠩ ⠦⠎⠊⠝ ⠨⠹⠸⠴ ⠸⠱ ⠊⠎ ⠏⠗⠕⠝⠳⠝⠉⠫ ⠦⠎⠔⠑
⠮⠞⠁⠴⠲

Other spacing rules of the Nemeth Code may take precedence.

Example 14.2-2 For any angle θ, $\sin(\theta+360°)=\sin\theta$ and $\cos(\theta+360°)=\cos\theta$.

⠠⠿ ⠁⠝⠽ ⠁⠝⠛⠇⠑ ⠸⠩ ⠨⠹⠠
⠎⠊⠝ ⠷⠨⠹⠬⠒⠖⠴⠘⠨⠡⠐⠾ ⠨⠅ ⠎⠊⠝ ⠨⠹ ⠠⠄⠯
⠉⠕⠎ ⠷⠨⠹⠬⠒⠖⠴⠘⠨⠡⠐⠾ ⠨⠅ ⠉⠕⠎ ⠨⠹ ⠸⠱⠲

Within the expression, the space preceding "sin" and "cos" is present because a space is required following an equals sign. Note that each function name ("sin" and "cos") is followed by a space even though a space may not appear in print.

14.2.1 Spacing with Signs of Operation: Although a sign of operation is usually unspaced from the symbols which precede and follow it, the space required after a function name takes precedence.

⋙ $\tan\cdot\sin$

⠞⠁⠝ ⠡⠎⠊⠝

"tan" is followed by a space. (A space is not required before "sin".)

⋙ $1/\cos-\cos=\tan\cdot\sin$

⠼⠂⠸⠌⠉⠕⠎ ⠤⠉⠕⠎ ⠨⠅ ⠞⠁⠝ ⠡⠎⠊⠝

Each function name is followed by a space. The only function name preceded by a space is the one following the equals sign ("tan") because a space is required following a sign of comparison.

14.2.2 Spacing with Indicators: A space is not inserted between a function name and an indicator which applies to it.

⋙ $\frac{1}{\cos}$

⠹⠂⠌⠉⠕⠎⠼

There is no space between "cos" and the closing fraction indicator.

Example 14.2-3 **Reciprocal Functions:** $\frac{1}{\cos}-\cos=\tan\cdot\sin$

⠘⠂⠠⠗⠑⠉⠊⠏⠗⠕⠉⠁⠇ ⠘⠂⠠⠋⠥⠝⠉⠰⠝⠎⠒
⠸⠩ ⠹⠂⠌⠉⠕⠎⠼⠤⠉⠕⠎ ⠨⠅ ⠞⠁⠝ ⠡⠎⠊⠝ ⠸⠱

Note that the "required space" following the first function name "cos" is not present here because the closing fraction indicator applies to it. The minus sign is unspaced from the closing fraction indicator because spacing before and after a fraction is subject to the spacing rules for the signs preceding and following the fraction. (Review ***Lesson 9****.)*

Examples 14.2-4-14 Examine the spacing in the following examples. Also notice that mathematical letters are printed in italics, and abbreviated function names are in regular type.

(4) $\cos\theta = \frac{1}{\sin\theta}$

⠉⠕⠎ ⠨⠹ ⠨⠅ ⠹⠂⠌⠎⠊⠝ ⠨⠹⠼

(5) $\sin(\theta + 90°) = \cos\theta$

⠎⠊⠝ ⠷⠨⠹⠬⠔⠴⠘⠨⠡⠐⠾ ⠨⠅ ⠉⠕⠎ ⠨⠹

(6) $y = 3\tan 2x$

⠽ ⠨⠅ ⠼⠒⠞⠁⠝ ⠼⠆⠭

(7) $a^2 = 2ac\cos\beta + c^2$

⠁⠘⠆ ⠨⠅ ⠼⠆⠁⠉⠉⠕⠎ ⠨⠃⠬⠉⠘⠆

(8) $\sin(\alpha + \beta) + \sin(\alpha - \beta)$

⠎⠊⠝ ⠷⠨⠁⠬⠨⠃⠾⠬⠎⠊⠝ ⠷⠨⠁⠤⠨⠃⠾

(9) $y' = x\cos\varphi - y\sin\varphi$

⠽⠄ ⠨⠅ ⠭⠉⠕⠎ ⠨⠋⠤⠽⠎⠊⠝ ⠨⠋

(10) $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} = C$

⠼⠆⠜⠭⠻⠎⠊⠝ ⠜⠭⠻⠬⠆⠉⠕⠎ ⠜⠭⠻ ⠨⠅ ⠠⠉

(11) $6\sin 2A\cos 4A$

⠼⠖⠎⠊⠝ ⠼⠆⠠⠁⠉⠕⠎ ⠼⠲⠠⠁

(12) $\cos 203°\csc 203°$

⠉⠕⠎ ⠼⠆⠴⠒⠘⠨⠡⠐⠉⠎⠉ ⠼⠆⠴⠒⠘⠨⠡

(13) $\dfrac{2\sin\frac{\alpha}{2}}{2\cos\frac{\alpha}{2}}$

⠠⠹⠆⠎⠊⠝ ⠹⠨⠁⠌⠆⠼⠠⠌⠆⠉⠕⠎ ⠹⠨⠁⠌⠆⠼⠠⠼

(14) $\ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

⠇⠝ ⠳⠞⠁⠝ ⠷⠹⠨⠏⠌⠲⠼⠬⠹⠭⠌⠆⠼⠾⠳⠬⠠⠉

"ln" is the abbreviated function name for "natural logarithm". (Refer to the table of function names at the beginning of this lesson.)

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(1) $\sin 30° \cos 45°$

(2) $\sin\theta / \cos\theta$

(3) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

(4) $\dfrac{\tan 90°}{\cot 90°}$

(5) $r[3\cos\theta + 4\sin\theta] = 5$

(6) $7(\cos 20° + i\sin 20°)$

(7) $\frac{1}{2}\ln|\sec 2t + \tan 2t| + C$

(8) $a\sin\frac{x}{a}\cdot\frac{1}{a} = \sin\frac{x}{a}$

14.3 Nonuse of the English Letter Indicator: The English letter indicator is not used with an English letter following a function name.

⫸ sin x ⠎⠊⠝⠀⠭

The same rule applies to a letter combination that mimics a shortform, or to a Roman numeral following a function name. No ELI is used.

⫸ vers ab ⠧⠑⠗⠎⠀⠁⠃

⫸ cot I ⠉⠕⠞⠀⠠⠊

Examples 14.3-1-6 Examine the English letters and the spacing in the following examples.

(1) $\sin x + y$
⠎⠊⠝⠀⠭⠬⠽

(2) $\text{ctn} - \text{A} = -\text{ctn A}$
⠉⠞⠝⠀⠤⠠⠁⠀⠨⠅⠀⠤⠉⠞⠝⠀⠠⠁

(3) $y = 2\sin x + \sin 2x$
⠽⠀⠨⠅⠀⠼⠆⠎⠊⠝⠀⠭⠬⠎⠊⠝⠀⠼⠆⠭

(4) $y = \sqrt{\cot x}$
⠽⠀⠨⠅⠀⠜⠉⠕⠞⠀⠭⠻

(5) $\{\sin x \mid \sin x + 2 \leq +1\}$
⠨⠷⠎⠊⠝⠀⠭⠀⠳⠀⠎⠊⠝⠀⠭⠬⠆⠀⠐⠅⠱⠀⠬⠂⠨⠾

(6) $y = \ln|\tan x|$
⠽⠀⠨⠅⠀⠇⠝⠀⠳⠞⠁⠝⠀⠭⠳

14.4 Unabbreviated Function Names in Mathematical Context: When an unabbreviated function name occurs in mathematical context it is brailled in Nemeth Code, without contractions. "Mathematical context" includes being associated with a mathematical symbol ...

⫸ Sine I ⠠⠎⠊⠝⠑⠀⠠⠊

This unabbreviated function name ("Sine") is in Nemeth Code because it is associated with the Roman numeral "I".

⫸ $\text{sine}\, x - \text{sine}\, y$ ⠎⠊⠝⠑⠀⠭⠤⠎⠊⠝⠑⠀⠽

... or when used in conjunction with an abbreviated function name.

Example 14.4-1 What is the meaning of logsine?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠍⠂⠝⠬ ⠷ ⠸⠩ ⠇⠕⠛⠎⠊⠝⠑ ⠸⠱⠦

Abbreviated function names are mathematical material. "log" is an abbreviated function name, therefore "logsine" requires a switch to Nemeth Code.

Example 14.4-2 The logsine function is related to the logcosine function by

$$S_n = 2C_n.$$

⠠⠮ ⠸⠩ ⠇⠕⠛⠎⠊⠝⠑ ⠸⠱ ⠋⠥⠝⠉⠰⠝ ⠊⠎ ⠗⠑⠇⠁⠞⠫ ⠞⠕
⠮ ⠸⠩ ⠇⠕⠛⠉⠕⠎⠊⠝⠑ ⠸⠱ ⠋⠥⠝⠉⠰⠝ ⠃⠽
⠸⠩ ⠠⠎⠰⠝ ⠨⠅ ⠼⠆⠠⠉⠰⠝ ⠸⠱⠲

14.5 Consecutive Function Names: A space is required between two or more consecutive function names unless they are clearly unspaced in the print text. When there is doubt concerning the presence of a space between the function names or their abbreviations, a space should be left in the transcription.

⫸ $y = \text{arc}\ \sin x$ ⠽ ⠨⠅ ⠁⠗⠉ ⠎⠊⠝ ⠭

Example 14.5-1 What is cos arctan (-1)?

⠠⠱⠁⠞ ⠊⠎ ⠸⠩ ⠉⠕⠎ ⠁⠗⠉⠞⠁⠝ ⠷⠤⠂⠾ ⠸⠱⠦

Examples 14.5-2-6 Study the following examples.

(2) $n = \log \sin 50°27'$

⠝ ⠨⠅ ⠇⠕⠛ ⠎⠊⠝ ⠼⠢⠴⠘⠨⠡⠐⠆⠶⠄

(3) $\cos\left[2\ \text{Arc}\ \text{csc}\left(-\frac{7}{5}\right)\right]$

⠉⠕⠎ ⠈⠷⠆⠠⠁⠗⠉ ⠉⠎⠉ ⠷⠤⠹⠶⠌⠢⠼⠾⠈⠾

(4) $\cos\left(\text{arc}\ \text{Tan}\, x + \frac{\pi}{3}\right)$

⠉⠕⠎ ⠷⠁⠗⠉ ⠠⠞⠁⠝ ⠭⠬⠹⠨⠏⠌⠒⠼⠾

(5) Arc Sine x

⠠⠁⠗⠉ ⠠⠎⠊⠝⠑ ⠭

(6) $\text{Arctan}\, x + \text{Arccot}\, x = \frac{1}{2}\pi$

⠠⠁⠗⠉⠞⠁⠝ ⠭⠬⠠⠁⠗⠉⠉⠕⠞ ⠭ ⠨⠅ ⠹⠂⠌⠆⠼⠨⠏

14.5.1 "Arc" in Context: "Arc" can be an unabbreviated function name as well as the abbreviated function name. Transcribe this function name in the braille code that the associated material is in.

⫸ arc θ ⠁⠗⠉ ⠨⠹

The function name "arc" is brailled in Nemeth Code because it is associated with a math symbol.

Example 14.5-7 What is the arc sine function?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠜⠉ ⠎⠔⠑ ⠋⠥⠝⠉⠰⠝⠦

"sine" is unabbreviated (UEB), so "arc" is also brailled in UEB.

Example 14.5-8 What is the arc sin function?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠸⠩ ⠁⠗⠉ ⠎⠊⠝ ⠸⠱ ⠋⠥⠝⠉⠰⠝⠦

"sin" is abbreviated (Nemeth Code), so "arc" is also brailled in Nemeth Code.

14.5.1.a Do Not Confuse: The word "arc" can also refer to a curved line.

Example 14.5-9 Arc ACB is a major arc in Circle O.

⠠⠜⠉ ⠸⠩ ⠠⠁⠠⠉⠠⠃ ⠸⠱ ⠊⠎ ⠁ ⠍⠁⠚⠕⠗ ⠜⠉ ⠔
⠠⠉⠊⠗⠉⠇⠑ ⠠⠕⠲

"Arc" is not a function name in this context. It is referring to the curved line, "ACB".

14.6 Punctuation: An abbreviated function name is a mathematical expression and is punctuated accordingly. An unabbreviated function name is punctuated in the mathematical or literary mode according to its context.

Example 14.6-1 Some trigonometric functions are sine, cosine, and tangent.

⠠⠐⠎ ⠞⠗⠊⠛⠕⠝⠕⠍⠑⠞⠗⠊⠉ ⠋⠥⠝⠉⠰⠝⠎ ⠜⠑ ⠎⠔⠑⠂
⠉⠕⠎⠔⠑⠂ ⠯ ⠞⠁⠝⠛⠢⠞⠲

Unabbreviated function names are punctuated in literary mode, in UEB.

Example 14.6-2 Some trigonometric functions are sin, cos, and tan.

⠠⠐⠎ ⠞⠗⠊⠛⠕⠝⠕⠍⠑⠞⠗⠊⠉ ⠋⠥⠝⠉⠰⠝⠎ ⠜⠑ ⠸⠩ ⠎⠊⠝⠠
⠉⠕⠎⠠ ⠠⠄⠯ ⠞⠁⠝ ⠸⠱⠲

Abbreviated function names are punctuated in mathematical mode, in Nemeth Code.

Example 14.6-3 "Arcsin" is the "inverse sine."

⠸⠩ ⠦⠠⠁⠗⠉⠎⠊⠝⠸⠴ ⠸⠱ ⠊⠎ ⠮ ⠦⠔⠧⠻⠎⠑ ⠎⠔⠑⠲⠴

"Arcsin" is an abbreviated function name (math); "inverse sine" is an unabbreviated function name (narrative).

14.7 Keep Together: A function name and the sign which follows it is regarded as a single mathematical item and therefore should not be divided between braille lines.

Example 14.7-1 If θ = 51° is the angle between vectors, determine sin θ and cos θ.

⠠⠊⠋ ⠸⠩ ⠨⠹ ⠨⠅ ⠼⠢⠂⠘⠨⠡ ⠸⠱ ⠊⠎ ⠮ ⠁⠝⠛⠇⠑ ⠆⠞
⠧⠑⠉⠞⠕⠗⠎⠂ ⠙⠑⠞⠻⠍⠔⠑ ⠸⠩ ⠎⠊⠝ ⠨⠹ ⠠⠄⠯ ⠀⠀⠀⠀⠀⠀⠀
⠉⠕⠎ ⠨⠹ ⠸⠱⠲

"cos θ" is not divided between lines even though "cos" fits on the previous line.

The rule applies even in UEB context, illustrated in this example of spaced consecutive function names.

Example 14.7-2 ArcTan[z] gives the arc tangent of the complex number z.

⠸⠩ ⠠⠁⠗⠉⠠⠞⠁⠝ ⠈⠷⠵⠈⠾ ⠸⠱ ⠛⠊⠧⠑⠎ ⠮ ⠀⠀⠀⠀⠀⠀⠀
⠜⠉ ⠞⠁⠝⠛⠢⠞ ⠷ ⠮ ⠉⠕⠍⠏⠇⠑⠭ ⠝⠥⠍⠃⠻ ⠰⠵⠲

"arc tangent" is not divided between lines even though "arc" fits on the previous line.

14.8 CLARIFICATION—Function Names in an Enclosed List: Although the numeric indicator is not used at the beginning of an item in an enclosed list, it must be used before a numeral (or decimal point and a numeral) following a function name.

⫸ (2sin 30°, 3cos 60°)

⠷⠆⠎⠊⠝ ⠼⠒⠴⠘⠨⠡⠠ ⠒⠉⠕⠎ ⠼⠖⠴⠘⠨⠡⠐⠾

A function name and the item which follows it are regarded as a single item.

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(A) sine $x -$ sine y

(B) $2\sin x + 3\cos y$

(C) $\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x}$

(D) The logarithm of sine 18° is written log sin 18°.

(E) $\cos 225° = -\sqrt{\frac{1+\cos 450°}{2}}$

(F) ArcTan[x, y] gives the arc tangent of $\frac{y}{x}$, taking into account which quadrant the point (x, y) is in.

(G) The arc tangent of the complex number q is written "ArcTan[q]".

(H) Consider the ordered pair (cos .8000, 2 cos .8000).

14.9 Function Names and Superscripts/Subscripts: When a function name carries a superscript or a subscript, the required space follows the superscript or subscript. A letter, numeral, or other mathematical expression following this space assumes the same level as the function name. (In the following three examples, letters "θ" and "x" are printed on the baseline of writing and are unspaced.)

⋙ $\sin^2\theta$ ⠎⠊⠝⠘⠆ ⠨⠹

⋙ $\sin^2 x$ ⠎⠊⠝⠘⠆ ⠭

⋙ $\text{sine}^2 x$ ⠎⠊⠝⠑⠘⠆ ⠭

Example 14.9-1 The coordinates are the cosine and sine, so we conclude $\sin^2\theta + \cos^2\theta = 1$.

⠠⠮ ⠉⠕⠕⠗⠙⠔⠁⠞⠑⠎ ⠜⠑ ⠮ ⠉⠕⠎⠔⠑ ⠯ ⠎⠔⠑⠂ ⠎ ⠺⠑

⠒⠉⠇⠥⠙⠑ ⠸⠩ ⠎⠊⠝⠘⠆ ⠨⠹⠬⠉⠕⠎⠘⠆ ⠨⠹ ⠨⠅ ⠼⠂ ⠸⠱⠲

The required space follows the superscript in $\sin^2$ *and* $\cos^2$*. There is no space following the plus sign.*

Example 14.9-2 Verify that $1-\cos\frac{2\pi}{3} = 2\sin^2\frac{4\pi}{3}$.

⠠⠧⠻⠊⠋⠽ ⠞ ⠸⠩

⠼⠂⠤⠉⠕⠎ ⠹⠆⠨⠏⠌⠒⠼ ⠨⠅ ⠼⠆⠎⠊⠝⠘⠆ ⠹⠲⠨⠏⠌⠒⠼ ⠸⠱⠲

The required space follows "cos" and "$\sin^2$*". There is no space following the minus sign.*

Examples 14.9-3-6 Study the following examples.

(3) $\sin^2 A + \cos^2(B+A)$

⠎⠊⠝⠘⠆ ⠠⠁⠬⠉⠕⠎⠘⠆ ⠷⠠⠃⠬⠠⠁⠾

(4) $(1-\sin^2 x)^2\cos^2 x$

⠷⠂⠤⠎⠊⠝⠘⠆ ⠭⠾⠘⠆⠐⠉⠕⠎⠘⠆ ⠭

(5) $\sin^2\theta\times\frac{\cos^2\theta}{\sin^2\theta}-1$

⠎⠊⠝⠘⠆ ⠨⠹⠈⠡⠹⠉⠕⠎⠘⠆ ⠨⠹⠌⠎⠊⠝⠘⠆ ⠨⠹⠼⠤⠂

(6) $\dfrac{1-\dfrac{\sin^2 x}{\cos^2 x}}{\sec^2 x}$

⠠⠹⠂⠤⠹⠎⠊⠝⠘⠆ ⠭⠌⠉⠕⠎⠘⠆ ⠭⠼⠠⠌⠎⠑⠉⠘⠆ ⠭⠠⠼

14.9.1 Use/Nonuse of the Subscript Indicator: The subscript indicator is not used when an abbreviated function name carries a numeric subscript on the first level below the baseline of writing.

⫸ $\log_3 81 = 4$ ⠇⠕⠛⠒ ⠼⠦⠂ ⠨⠅ ⠼⠲

The numeral "3" is printed at the subscript level.

A subscript indicator is required in all other circumstances, including an unabbreviated function name which carries a numeric subscript.

⫸ $\text{logarithm}_3\ 81 = 4$ ⠇⠕⠛⠁⠗⠊⠞⠓⠍⠰⠒ ⠼⠦⠂ ⠨⠅ ⠼⠲

A subscript indicator is required because the subscript applies to a word (an unabbreviated function name is a word).

⫸ $\log_b N \cdot \log_a b$ ⠇⠕⠛⠰⠃ ⠠⠝⠡⠇⠕⠛⠰⠁ ⠃

A subscript indicator is required because each subscript is a letter.

⫸ $\log_{2e} x = -1.4$ ⠇⠕⠛⠰⠆⠑ ⠭ ⠨⠅ ⠤⠼⠂⠨⠲

A subscript indicator is required because the subscript contains a letter.

14.9.2 Function Names Within a Superscript or a Subscript: When a function name occurs within a superscript or subscript, the required space following it maintains the level at which the function name appears. A restatement of the level indicator is not needed.

⫸ $e^{\sin x}$ ⠑⠘⠎⠊⠝ ⠭

"sin x" is in the superscript position. Because the item immediately following the space after a function name assumes the same level as its related function name, no level indicator is needed for "x".

⫸ $y = e^{\cos^2 x}$ ⠽ ⠨⠅ ⠑⠘⠉⠕⠎⠘⠘⠆ ⠭

"cos^2 x" is in the superscript position. Because the item immediately following the space after a function name assumes the same level as its related function name, no level indicator is needed for "x".

Examples 14.9-7-15 Study the following examples.

(7) $y = e^{\sin x}$

⠽ ⠨⠅ ⠑⠘⠎⠊⠝ ⠭

(8) $y = e^{\text{sine}\, x}$

⠽ ⠨⠅ ⠑⠘⠎⠊⠝⠑ ⠭

(9) $y = (\sin x)^{\tan x}$

⠽ ⠨⠅ ⠷⠎⠊⠝ ⠭⠾⠘⠞⠁⠝ ⠭

(10) $e^{\ln x - 2\ln y}$

⠑⠘⠇⠝ ⠭⠤⠆⠇⠝ ⠽

(11) $\mathrm{a}^{\mathrm{g(x)log_a f(x)}}$

⠁⠘⠛⠷⠭⠾⠇⠕⠛⠘⠰⠁ ⠋⠷⠭⠾

(12) $3^{\log_3 9}$

⠼⠒⠘⠇⠕⠛⠘⠰⠒ ⠼⠔

*Recall from **Lesson 8** that a subscript indicator is required in superscript and subscript combinations. The super/sub indicator shows a numeric subscript in the superscript position.*

(13) $3^{\log_3 7} + 2^{\log_2 5}$

⠼⠒⠘⠇⠕⠛⠘⠰⠒ ⠼⠶⠐⠬⠆⠘⠇⠕⠛⠘⠰⠆ ⠼⠢

See note above.

(14) $a^{\log_a x} = x$

⠁⠘⠇⠕⠛⠘⠰⠁ ⠭ ⠨⠅ ⠭

*Recall from **Lesson 8** that the space before a comparison sign returns the reader to the baseline.*

(15) $e^{\sin x = a} > y$

⠑⠘⠎⠊⠝ ⠭ ⠘⠨⠅ ⠁ ⠨⠂ ⠽

*Recall from **Lesson 8** that when a comparison sign occurs within a superscript, the level is restated before the comparison sign.*

PRACTICE 14C

1. $\log_n .125 = -.6$
2. $\mathrm{antilog_a\, x = N}$
3. $\mathrm{log_{.0543} x} = -.7$
4. $\cot^{-1} x + \frac{\pi}{2} - \tan^{-1} x$
5. $\sin^2 90° + \cos^2 90° = 1$
6. $e^{x + \ln x}$
7. $e^{\sin x} + e^{\sin y}$
8. $2^{\sec x} = y$

14.10 Modifiers with Function Names: Modified function names are transcribed according to the five-step rule for the transcription of modified expressions introduced in **Lesson 12**.

⫸ $\lim\limits_{x \to a}$ ⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠁⠻

When a function name carries a modifier, the required space after the function name follows the termination of the modifier.

⫸ $\lim\limits_{x\to a} f(x) = 1$ ⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠁⠻ ⠋⠷⠭⠾ ⠨⠅ ⠼⠂

Examples 14.10-1-2 Study these additional examples.

(1) $\lim\limits_{x \uparrow 4}(x - 4)^{-1}$

⠐⠇⠊⠍⠩⠭ ⠫⠣⠒⠒⠕ ⠼⠲⠻ ⠷⠭⠤⠲⠾⠘⠤⠂

(2) $\lim\limits_{\theta \longrightarrow \theta_0} (\tan \theta)$

⠐⠇⠊⠍⠩⠨⠹ ⠫⠕ ⠨⠹⠴⠻ ⠷⠞⠁⠝ ⠨⠹⠾

14.10.1 SPECIAL CASE—Upper Limit and Lower Limit: The symbols below denote "upper limit" or "lower limit". The horizontal bar directly over or under the function name "lim" or `limit" is not treated as a modifier.

$\overline{\text{lim}}$ or $\overline{\text{limit}}$	⠣⠇⠊⠍	or	⠣⠇⠊⠍⠊⠞	upper limit

⫸ $\overline{\lim\limits_{n\to\infty}} f_n(x)$ ⠐⠣⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

⫸ $\overline{\text{limit}}_{n \to \infty} f_n(x)$ ⠐⠣⠇⠊⠍⠊⠞⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

$\underline{\text{lim}}$ or $\underline{\text{limit}}$	⠩⠇⠊⠍	or	⠩⠇⠊⠍⠊⠞	lower limit

⫸ $\underline{\lim}_{n\to\infty} f_n(x)$ ⠐⠩⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

⫸ $\underline{\text{limit}}_{n \to \infty} f_n(x)$ ⠐⠩⠇⠊⠍⠊⠞⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

PRACTICE 14D

1. Find $\lim\limits_{x \to 0.6} 2^{25x^2-10x-1}$.
2. Formulate a precise definition for $\lim\limits_{x \downarrow -\infty} f(x) = L$.
3. If $\overline{\lim\limits_{n\to\infty}}\, a_n = A$ and $\overline{\lim\limits_{n\to\infty}}\, b_n = B$, must it be true that $\overline{\lim\limits_{n\to\infty}}(a_n + b_n) = A + B$?
4. Find $\overline{\operatorname*{limit}_{n\to\infty}}\, a_n$ when $a_n = (-1)_n$.
5. $\lim\limits_{x\to 0} \csc x \ln(1 + x)$

FORMAT

Division of Mathematical Expressions Between Braille Lines

14.11 Review: Recall from **Lesson 2** that a mathematical expression must not be divided between lines if it will fit on one braille line within the current margins. If there is insufficient space on the remainder of a line, the entire expression is brought down to the next line. In this lesson we will discuss what to do when a mathematical expression is too long to be contained within the current margins – that is, when a division is unavoidable. First, here is a review of "won't fit " topics already covered.

- **Nemeth Code Switch Indicators** (Lessons 2 and 4) If both switch indicators will not fit on the same line as the math expression, the opening Nemeth Code indicator may fall on the previous line. The Nemeth Code terminator and any related punctuation may be placed on the following line. **2.5.2, 4.7**
- **Long Numeral** (Lesson 2) A long numeral which cannot be contained on one braille line within the current margins is divided after a comma if a comma is present, and a hyphen is inserted. If the numeral does not contain a comma, the hyphen may be inserted after any digit. When a numeral is divided between braille lines, the numeric indicator is restated before the first digit of the numeral on the next line. Take another look at the two examples in **2.7.1.**
- **Enclosed List:** (Lesson 5) If an enclosed list will not fit on a single braille line, a runover line can begin following a comma used to separate the items. Take another look at the examples in **5.15.3.a.**
- **Linked Expressions** (Lesson 9) If a linked expression is too long to fit on one line, the expression continues on the next line, beginning with a sign of comparison. It is not necessary to divide at every comparison sign. The new line begins in the runover cell of the current format. **9.22**
- **Keystrokes** (Lesson 11) If items in a string of keyboard or calculator keystrokes will not fit on a single braille line, division may be made after any item in the keystroke string, but not within the keystroke. An example is shown in **11.25**.

14.12 The Concept of Logical Mathematical Units: When a math expression is too long to fit on one braille line, the expression is divided according to a priority list given in the Nemeth Code. When the rule is properly applied, a long or complicated mathematical expression will be disassembled into a series of logical mathematical units allowing the reader to mentally reassemble the expression. Conversely, a poorly divided expression will hinder the reader's understanding of the mathematics. Shrewd application of this rule can be properly rendered even if the transcriber is unfamiliar with the particular mathematics.

14.12.1 The Priority List: Mathematical expressions such as formulas, equations, etc., which cannot fit on one braille line within the boundaries of the margins used are to be divided between lines in the following order of preference:

- before a comparison sign on the baseline of writing;
- before an operation sign on the baseline of writing;
- before a fraction line;
- before a baseline indicator;
- before a level indicator;
- between factors;
- after a termination indicator.

The examples in this section do not show code switch indicators. Assume mathematical context throughout. Full margins are shown for embedded material. Displayed material and subdivisions begin in cell 3 with runovers in cell 5. Reminder: A 38-cell line is used for the examples in this book.

14.13 PRIORITY #1—Before a Sign of Comparison: A division may be made before a sign of comparison as long as the sign is on the baseline of writing.

Example 14.13-1 $1778 + 1294 + 865 + 905 + 2574 + 485 + 100\ >\ 8000$

Embedded:

⠼⠂⠶⠶⠦⠬⠂⠆⠔⠲⠬⠦⠖⠢⠬⠔⠴⠢⠬⠆⠢⠶⠲⠬⠲⠦⠢⠬⠂⠴⠴⠀⠀⠀⠀⠀⠀⠀
⠨⠂ ⠼⠦⠴⠴⠴

The expression continues on the next line, beginning with the sign of comparison.

Displayed:

⠼⠂⠶⠶⠦⠬⠂⠆⠔⠲⠬⠦⠖⠢⠬⠔⠴⠢⠬⠆⠢⠶⠲⠬⠲⠦⠢⠬⠂⠴⠴⠀⠀⠀⠀⠀
⠨⠂ ⠼⠦⠴⠴⠴

The new line begins in the runover cell of the current format.

Note that the print copy may divide after a comparison sign, but the braille transcription follows Nemeth Code rules and divides before the comparison sign.

Example 14.13-2 $(1778 + 1294 + 865) + (905 + 2574 + 485) + 100\ =$
$3937 + 3964 + 100\ =\ 8001$

⠷⠂⠶⠶⠦⠬⠂⠆⠔⠲⠬⠦⠖⠢⠾⠬⠷⠔⠴⠢⠬⠆⠢⠶⠲⠬⠲⠦⠢⠾⠬⠂⠴⠴⠀⠀⠀⠀
⠨⠅ ⠼⠒⠔⠒⠶⠬⠒⠔⠖⠲⠬⠂⠴⠴ ⠨⠅ ⠼⠦⠴⠴⠂

It is not necessary to divide at every comparison sign.

The transition to a new braille line made before a sign of comparison terminates the effect of any level indicator used on the line above, just as it would if the expression were not divided between lines.

Example 14.13-3 **a.** $9x^4 + 81y^4 = (3x^2)^2 + (9y^2)^2$

In a subdivision:

⠀⠀⠰⠁⠸⠲ ⠼⠔⠭⠘⠲⠐⠬⠦⠂⠽⠘⠲
⠨⠅ ⠷⠒⠭⠘⠆⠐⠾⠘⠆⠐⠬⠷⠔⠽⠘⠆⠐⠾⠘⠆

Continue the expression in the appropriate runover cell—cell 5, in this case.

14.13.1 Identifier May Stand Alone: Do not divide an expression if it will fit on one line. The identifier can stand alone on the previous line if the expression will not fit on the same line with the identifier.

Example 14.13-4 (1) $9x^4 + 81y^4 = (3x^2)^2 + (9y^2)^2$

In a main level entry starting in cell 1:

⠷⠂⠾
⠼⠔⠭⠘⠲⠐⠬⠦⠂⠽⠘⠲ ⠨⠅ ⠷⠒⠭⠘⠆⠐⠾⠘⠆⠐⠬⠷⠔⠽⠘⠆⠐⠾⠘⠆

Begin the expression in the appropriate runover cell—cell 3, in this case.

Proper margins are used as runover sites are applied.

Example 14.13-5 To factor the expression $-2ab + a^2 + b^2$,

Jared wrote: $-2ab + a^2 + b^2 = b^2 - 2ab + a^2 = (b - a)^2$

Dom wrote: $-2ab + a^2 + b^2 = a^2 - 2ab + b^2 = (a - b)^2$

Both are correct. Explain.

1 ⠠⠞⠕ ⠋⠁⠉⠞⠕⠗ ⠮ ⠑⠭⠏⠗⠑⠎⠨⠝
2 ⠸⠩ ⠤⠼⠆⠁⠃⠬⠁⠘⠆⠐⠬⠃⠘⠆ ⠸⠱⠂
3 ⠠⠚⠜⠫ ⠺⠗⠕⠞⠑⠒ ⠸⠩ ⠤⠼⠆⠁⠃⠬⠁⠘⠆⠐⠬⠃⠘⠆
4 ⠨⠅ ⠃⠘⠆⠐⠤⠆⠁⠃⠬⠁⠘⠆ ⠨⠅ ⠷⠃⠤⠁⠾⠘⠆ ⠸⠱
5 ⠠⠙⠕⠍ ⠺⠗⠕⠞⠑⠒ ⠸⠩ ⠤⠼⠆⠁⠃⠬⠁⠘⠆⠐⠬⠃⠘⠆
6 ⠨⠅ ⠁⠘⠆⠐⠤⠆⠁⠃⠬⠃⠘⠆ ⠨⠅ ⠷⠁⠤⠃⠾⠘⠆ ⠸⠱
7 ⠠⠃⠕⠹ ⠜⠑ ⠉⠕⠗⠗⠑⠉⠞⠲ ⠠⠑⠭⠏⠇⠁⠔⠲

Lines 1-2: New paragraph begins in cell 3 with runovers in cell 1.
Lines 3-6: Two displayed expressions – each begins in cell 3, with runover in cell 5.
Line 7: Paragraph continues in the runover cell (cell 1).

14.13.2 *Logical Mathematical Units*—A comparison sign within a modifier, superscript, subscript, fraction, radical expression, etc. is not a logical division site.

Example 14.13-6

$$\sum_{i=0}^{n}\sum_{j=0}^{m} = \lim_{n\to\infty} \underline{\hspace{3em}}$$

⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠴⠣⠝⠻⠐⠨⠠⠎⠩⠚ ⠨⠅ ⠼⠴⠣⠍⠻⠀⠀⠀⠀⠀⠀⠀⠀
⠨⠅ ⠐⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠤⠤⠤⠤

Only the comparison sign on the baseline is an appropriate division site.

Also note that material within mathematical grouping signs is a unit and should not be divided unless the entire grouping will not fit on one line.

Example 14.13-7 {x | x is an even integer} ⊈ {x | x is an odd integer}

```
⠨⠷⠭ ⠳ ⠭ ⠊⠎ ⠁⠝ ⠑⠧⠑⠝ ⠊⠝⠞⠑⠛⠑⠗⠨⠾
⠌⠸⠐⠅⠱ ⠨⠷⠭ ⠳ ⠭ ⠊⠎ ⠁⠝ ⠕⠙⠙ ⠊⠝⠞⠑⠛⠑⠗⠨⠾
```

Although " ⊈ {x" will fit at the end of the first line, only the comparison sign outside of the grouped expressions is an appropriate division site.

If the expression will fit on one line, do not divide it. Sometimes this means starting the expression on a new line following an identifier. The new line begins in the appropriate runover cell.

Example 14.13-8 Dividing by $a^2(a^2 - c^2)$ we have

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1. \qquad (3.3)$$

```
1    ⠠⠙⠊⠧⠊⠙⠬ ⠃⠽ ⠸⠩ ⠁⠘⠆⠐⠷⠁⠘⠆⠐⠤⠉⠘⠆⠐⠾ ⠸⠱ ⠺⠑
2  ⠓ ⠸⠩
3    ⠷⠒⠨⠒⠾
4      ⠹⠭⠘⠆⠐⠌⠁⠘⠆⠐⠼⠬⠹⠽⠘⠆⠐⠌⠁⠘⠆⠐⠤⠉⠘⠆⠐⠼ ⠨⠅ ⠼⠂
5      ⠸⠱⠲
```

Lines 3-5: Displayed margins to narrative are 3-5.
Line 3: The label may sit alone on a line. The math will not fit on this line.
*(Review **Lesson 11** regarding displayed material with labels.)*
Line 4: The displayed expression will fit on one line in the runover position.
Line 5: The Nemeth Code terminator and punctuation may stand alone on a line.

14.14 PRIORITY #2—Before a Sign of Operation: A new line may begin before a sign of operation on the baseline of writing. Use as much of the current line as possible before dividing. It is not necessary to divide at every operation sign.

Example 14.14-1 $1\frac{5}{6} + 9\frac{1}{2} + 3\frac{1}{12} + 2\frac{1}{4} + 3\frac{1}{2}$

Embedded:

```
⠼⠂⠸⠹⠢⠌⠖⠸⠼⠬⠔⠸⠹⠂⠌⠆⠸⠼⠬⠒⠸⠹⠂⠌⠂⠆⠸⠼⠬⠆⠸⠹⠂⠌⠲⠸⠼
⠬⠒⠸⠹⠂⠌⠆⠸⠼
```

If the transition to a new line is made before a minus sign and a numeral, or before a minus sign and a decimal point and a numeral, a numeric indicator is required.

Example 14.14-2 $14x^3 - 15y^3 + 5x^2 - 4y^2 + 16x - 10y + 31$

Displayed:

⠀⠀⠼⠂⠲⠭⠘⠒⠐⠤⠂⠢⠽⠘⠒⠐⠬⠢⠭⠘⠆⠐⠤⠲⠽⠘⠆⠐⠬⠂⠖⠭
⠀⠀⠀⠀⠤⠼⠂⠴⠽⠬⠒⠂

14.14.1 *Logical Mathematical Units*—If the expression also contains a baseline sign of comparison, the "priority 1" division is applied first in order to maintain an orderly representation of logical mathematical units.

Example 14.14-3 $1\frac{5}{6} + 9\frac{1}{2} + 3\frac{1}{12} + 2\frac{1}{4} + 3\frac{1}{2} = 20\frac{1}{6}$

⠼⠂⠸⠹⠢⠌⠖⠸⠼⠬⠔⠸⠹⠂⠌⠆⠸⠼⠬⠒⠸⠹⠂⠌⠂⠆⠸⠼⠬⠆⠸⠹⠂⠌⠲⠸⠼
⠬⠒⠸⠹⠂⠌⠆⠸⠼
⠨⠅ ⠼⠆⠴⠸⠹⠂⠌⠖⠸⠼

The first division decision is to begin a new line with the equals sign (priority #1). Since the anchor does not fit on one braille line, it is divided before a plus sign (priority #2).

Apply this concept in order to maintain an orderly representation of logical mathematical units: If an expression must be divided at a site lower on the priority list, a new line is required at each sign which occurs higher on the list.

Example 14.14-4 $1\frac{5}{6} + 9\frac{1}{2} + 3\frac{1}{12} + 2\frac{1}{4} + 3\frac{1}{2} = 18\frac{26}{12} = 20\frac{2}{12} = 20\frac{1}{6}$

Embedded:

⠼⠂⠸⠹⠢⠌⠖⠸⠼⠬⠔⠸⠹⠂⠌⠆⠸⠼⠬⠒⠸⠹⠂⠌⠂⠆⠸⠼⠬⠆⠸⠹⠂⠌⠲⠸⠼
⠬⠒⠸⠹⠂⠌⠆⠸⠼
⠨⠅ ⠼⠂⠦⠸⠹⠆⠖⠌⠂⠆⠸⠼
⠨⠅ ⠼⠆⠴⠸⠹⠆⠌⠂⠆⠸⠼
⠨⠅ ⠼⠆⠴⠸⠹⠂⠌⠖⠸⠼

A new line is started at each sign which occurs higher on the list. Because the first link must be divided at a "priority 2" location (before an operation sign), every "priority 1" division site is also applied.

An operation sign within a modifier, superscript, subscript, fraction, radical expression, etc. is not a logical division site.

Example 14.14-5 $\frac{1}{y^2-6y+8}+\frac{1}{y^2-16}-\frac{5}{y^2+2y-8}=\frac{1}{y-4}$

Embedded:

⠹⠂⠌⠽⠘⠆⠐⠤⠖⠽⠬⠦⠼⠬⠹⠂⠌⠽⠘⠆⠐⠤⠂⠖⠼
⠤⠹⠢⠌⠽⠘⠆⠐⠬⠆⠽⠤⠦⠼
⠨⠅ ⠹⠂⠌⠽⠤⠲⠼

Only the operation signs on the baseline are appropriate division sites, not those in the denominators of the fractions. To maintain an orderly representation of logical mathematical units, the "priority 1" division (before the equals sign) is still applied.

Material within grouping signs is a unit whose division should be avoided (if possible).

Example 14.14-6 $(3\times 10^4)+(4\times 10^3)+(7\times 10^2)+(5\times 10)+(4\times 1)=N$

⠷⠒⠈⠡⠂⠴⠘⠲⠐⠾⠬⠷⠲⠈⠡⠂⠴⠘⠒⠐⠾⠬⠷⠶⠈⠡⠂⠴⠘⠆⠐⠾
⠬⠷⠢⠈⠡⠂⠴⠾⠬⠷⠲⠈⠡⠂⠾
⠨⠅ ⠠⠝

Only the operation signs (plus signs) outside of the grouped expressions are appropriate division sites, not those between the parentheses (the multiplication crosses).

Instructions: Keep the following points in mind as you select runover sites. (1) keep logical units intact on one line, if possible; (2) the highest priority available in the expression is utilized first.

PRACTICE 14E

A. $2\times 423=(2\times 400)+(2\times 20)+(2\times 3)=800+40+6=846$

B. $\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7=490^\circ$

C. $2\frac{3}{4}\text{ yd}+1\frac{3}{4}\text{ yd}+\frac{3}{4}\text{ yd}=5\frac{1}{4}\text{ yd}$

D. $\sqrt{(x+a^2)+(y+a^2)}-\sqrt{(x-a^2)+(y-a^2)}=\pm 2a$

Sommer's Routine: Sommer's routine can be represented by the following expression.

Routine S: $5+3+4(1+\ -1)+(-3)+(-5)+3\big(5+(-2)\big)+1$

Draw a simple diagram to represent *Routine S*.

CHALLENGE PROBLEM #1

The Nemeth Code lists "after a comma which occurs between items in an enclosed list" as the first item in the priority list. Since material within grouping signs is a unit whose division should be avoided, when an enclosed list is part of a larger math expression, division within the enclosed list should be made only if the entire list will not fit on one braille line. As seen below, dividing before the comparison sign has the desired effect of presenting the equation in logical units.

$$y = \left(-8, -4, \frac{1}{2}, 3, 8, 10, 20\frac{3}{4}, 100\right)$$

⠽
  ⠨⠅ ⠷⠤⠦⠠ ⠤⠲⠠ ⠹⠂⠌⠆⠼⠠ ⠒⠠ ⠦⠠ ⠂⠴⠠
  ⠆⠴⠸⠹⠒⠌⠲⠸⠼⠠ ⠂⠴⠴⠾

Since the expression will not fit on one line, a division is made before the equals sign.
The link will not fit on one line. The link is an enclosed list– it is divided after a comma.

CHALLENGE PROBLEM #2

Similarly, there is no need to separate the items in the enclosed list in the following anchor.

$$D(x, y, z) = \sqrt{(x-2)^2 + (y+1)^2 + (z-2)^2}$$

⠠⠙⠷⠭⠠ ⠽⠠ ⠵⠾
⠨⠅ ⠜⠷⠭⠤⠆⠾⠘⠆⠐⠬⠷⠽⠬⠂⠾⠘⠆⠐⠬⠷⠵⠤⠆⠾⠘⠆⠐⠻

Priority list so far:
— before a comparison sign on the baseline of writing;
— before an operation sign on the baseline of writing;

14.15 PRIORITY #3—Before a Fraction Line: A division may be made before a fraction line belonging to a fraction on the baseline of writing.

Example 14.15-1 $\dfrac{93{,}000\times 0.0006\times 2.0\times 10.56}{32\times 1.257\times 10}$

⠹⠔⠒⠠⠴⠴⠴⠈⠡⠴⠨⠴⠴⠴⠖⠈⠡⠆⠨⠴⠈⠡⠂⠴⠨⠢⠖
⠌⠒⠆⠈⠡⠂⠨⠆⠢⠶⠈⠡⠂⠴⠼

If the expression contains more than one such fraction line, use as much of the line as possible before dividing the expression. It is not necessary to divide at every fraction line.

14.15.1 *Logical Mathematical Units*—An expression that requires division before a fraction line must also be divided before every previous item on the priority list—that is, before every baseline comparison sign and every baseline operation sign. Avoid division within a numerator or within a denominator.

Example 14.15-2 $\dfrac{\text{Fixed Cost}}{\text{Unit Revenue}-\text{Marginal Cost}} = P$

⠹⠠⠋⠊⠭⠑⠙ ⠠⠉⠕⠎⠞
⠌⠠⠥⠝⠊⠞ ⠠⠗⠑⠧⠑⠝⠥⠑⠤⠠⠍⠁⠗⠛⠊⠝⠁⠇ ⠠⠉⠕⠎⠞⠼
⠨⠅ ⠠⠏

The entire fraction will not fit on one line, so a division is made at the fraction line.
Even though "= P" will fit on the second line (following the fraction's denominator), a division before the "priority 1" equals sign is required.

If the numerator or denominator requires a division, division at the fraction line must also be applied.

Example 14.15-3 $\dfrac{\dfrac{\mathrm{d}x}{\mathrm{d}t}\,\dfrac{\mathrm{d}^2y}{d\mathrm{t}^2} - \dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}\,\dfrac{\mathrm{d}y}{\mathrm{d}t}}{(\mathrm{d}x/\mathrm{d}t)^3}$

⠠⠹⠹⠙⠭⠌⠙⠞⠼⠹⠙⠘⠆⠐⠽⠌⠙⠞⠘⠆⠐⠼
⠤⠹⠙⠘⠆⠐⠭⠌⠙⠞⠘⠆⠐⠼⠹⠙⠽⠌⠙⠞⠼
⠠⠌⠷⠙⠭⠸⠌⠙⠞⠾⠘⠒⠐⠠⠼

The numerator will not fit on one braille line. A division is made before the minus sign.
A division is also required before the fraction line.
(Review complex fractions in ***Lesson 9****.)*

14.16 PRIORITY #4—Before a Baseline Indicator: If a baseline indicator is required to show a return to the baseline of writing after a superscript or subscript, it will be the first symbol on the new braille line of a divided expression. Use as much of the line as possible before dividing the expression.

Example 14.16-1 $49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2$

⠼⠲⠔⠭⠘⠶⠐⠽⠘⠖⠐⠤⠖⠒⠭⠘⠖⠐⠽⠘⠲⠐⠬⠢⠖⠭⠘⠢⠐⠽⠘⠢
⠐⠬⠖⠲⠭⠘⠲⠐⠽⠘⠆

The baseline indicator is the first symbol in the runover line.

Example 14.16-2 $\dfrac{e^{1/n}+e^{2/n}+e^{3/n}+\ldots+e^{(n-1)/n}+e^{n/n}}{n}$

⠹⠑⠘⠂⠸⠌⠝⠐⠬⠑⠘⠆⠸⠌⠝⠐⠬⠑⠘⠒⠸⠌⠝⠐⠬ ⠄⠄⠄
⠬⠑⠘⠷⠝⠤⠂⠾⠸⠌⠝⠐⠬⠑⠘⠝⠸⠌⠝
⠐⠌⠝⠼

A baseline indicator is needed before this fraction line, even when the symbol begins in the runover position on a new line of a divided expression.

Note that a baseline indicator is not inserted if otherwise not required.

Example 14.16-3 $a_1b_2c_3 + de_3f_1gh_2 + i_2j_3k_1 - lm_3no_2pq_1 - r_1s_3t_2 - uv_2wx_1yz_3$

⠁⠂⠃⠆⠉⠒⠬⠙⠑⠒⠋⠂⠛⠓⠆⠬⠊⠆⠚⠒⠅⠂⠤⠇⠍⠒⠝⠕⠆⠏⠟⠂
⠤⠗⠂⠎⠒⠞⠆⠤⠥⠧⠆⠺⠭⠂⠽⠵⠒

*Review **8.10** in Lesson 8 regarding nonuse of the subscript indicator.*

14.16.1 *Logical Mathematical Units*—An expression that requires division before a baseline indicator must also be divided before every previous item on the priority list.

Example 14.16-4 $(49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2) \div 7x^3y^2$

⠷⠲⠔⠭⠘⠶⠐⠽⠘⠖⠐⠤⠖⠒⠭⠘⠖⠐⠽⠘⠲⠐⠬⠢⠖⠭⠘⠢⠐⠽⠘⠢
⠐⠬⠖⠲⠭⠘⠲⠐⠽⠘⠆⠐⠾
⠨⠌⠶⠭⠘⠒⠐⠽⠘⠆

This expression must also begin a new line before the operation sign that is not within the enclosed expression (the division symbol) — a priority #2 site.

Instructions: Keep the following points in mind as you select runover sites. (1) keep logical units intact on one line, if possible; (2) the highest priority available in the expression is utilized first; (3) if necessary, choose additional sites as you descend the list.

PRACTICE 14F

Compute and/or simplify.

1. $\frac{(1.21\times10^4)(6.937\times10^8)}{3.75(10^2)}$

2. $\frac{\left(\frac{3}{2}\right)\times\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{1\times2\times3}$

3. $(\pm)a_{1i_1}a_{2i_2}a_{3i_3}a_{4i_4}\ldots\ a_{ni_n}$

4. $(16x^4+8x^3y+4x^2y^2+2xy^3+y^4)(2x-y)=32x^5-y^5$

5. $144\ \mathrm{ft}^2+107\ \mathrm{ft}^2+112\ \mathrm{ft}^2+145\ \mathrm{ft}^2-131\ \mathrm{ft}^2=N$

6. $\frac{\sqrt{0.4523}\times(2.6275)^3}{43690\times0.000982}$

7. $\pm\left(\frac{a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}}{3}\right)^{\frac{3}{2}}$

8. $\frac{1.793\ \mathrm{g\ Cu/g\ Cl}}{0.8964\ \mathrm{g\ Cu/g\ Cl}}=\underline{\qquad}$

Priority list so far:
- — before a comparison sign on the baseline of writing;
- — before an operation sign on the baseline of writing;
- — before a fraction line;
- — before a baseline indicator.

14.17 PRIORITY #5—Before a Level Indicator: A division may be made before a superscript or subscript indicator. The level indicator will be the first symbol in the runover line.

Example 14.17-1 $\frac{1}{2}\left[\frac{1}{2}\sin 2u - u\right]_{A(u=0)}^{P(u=u)}$

⠹⠂⠌⠆⠼⠈⠷⠹⠂⠌⠆⠼⠎⠊⠝ ⠼⠆⠥⠤⠥⠈⠾⠰⠠⠁⠷⠥ ⠰⠨⠅ ⠼⠴⠾
⠘⠠⠏⠷⠥ ⠘⠨⠅ ⠥⠾

Reminders: Keep the expression enclosed between brackets together on one line if possible.
The subscript is brailled first.

If transition to a new braille line must be made *within* a superscript or subscript, the level in effect continues when the division is made before a sign of operation or a fraction line, just as it would if the expression were not divided.

Example 14.17-2 $x^{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\,\ldots\,+\frac{1}{n}}$

⠭⠘⠂⠬⠹⠂⠌⠆⠼⠬⠹⠂⠌⠒⠼⠬⠹⠂⠌⠲⠼⠬⠹⠂⠌⠢⠼⠬⠹⠂⠌⠖⠼
⠬ ⠄⠄⠄ ⠬⠹⠂⠌⠝⠼

The plus sign beginning line 2 is still in the superscript position.

However, if the transition is made before a sign of comparison the level in effect is terminated (just as it would if the expression were not divided) and the level must be restated before the sign of comparison on the new braille line. [No example given.]

CHALLENGE PROBLEM #3

The relative humidity, *R*, is found using the following formula:

$$\mathrm{R} = 10^{\left(\frac{4221}{T+459.4} - \frac{4221}{D+459.4} + 2\right)}$$

where T is the air temperature (in °F) and D is the dew point temperature (in °F).

The superscript is too long to fit on one line.
Can you explain the division decisions made in the transcription?

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  ⠠⠮ ⠗⠑⠇⠁⠞⠊⠧⠑ ⠓⠥⠍⠊⠙⠰⠽⠂ ⠰⠠⠗⠂ ⠊⠎ ⠋⠨⠙ ⠥⠎⠬
⠮ ⠋⠕⠇⠇⠪⠬ ⠿⠍⠥⠇⠁⠒ ⠸⠩
  ⠠⠗
    ⠨⠅ ⠼⠂⠴
    ⠘⠷⠹⠲⠆⠆⠂⠌⠠⠞⠬⠲⠢⠔⠨⠲⠼⠤⠹⠲⠆⠆⠂⠌⠠⠙⠬⠲⠢⠔⠨⠲⠼
    ⠬⠆⠾ ⠸⠱
⠐⠱ ⠰⠠⠞ ⠊⠎ ⠮ ⠁⠊⠗ ⠞⠑⠍⠏⠻⠁⠞⠥⠗⠑ ⠐⠣⠔
⠸⠩ ⠘⠨⠡⠐⠠⠋ ⠸⠱⠐⠜ ⠯ ⠰⠠⠙ ⠊⠎ ⠮ ⠙⠑⠺ ⠏⠕⠔⠞
⠞⠑⠍⠏⠻⠁⠞⠥⠗⠑ ⠐⠣⠔ ⠸⠩ ⠘⠨⠡⠐⠠⠋ ⠸⠱⠐⠜⠲
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14.18 PRIORITY #6—Between Grouped Factors: A division may be made between factors that are enclosed within grouping signs. Use as much of the line as possible before dividing the expression. Begin a new line with the opening enclosure symbol of the next factor.

Example 14.18-1 $(3x^2y)(3x^2y)(3x^2y)(3x^2y)(3x^2y)$

⠷⠒⠭⠘⠆⠐⠽⠾⠷⠒⠭⠘⠆⠐⠽⠾⠷⠒⠭⠘⠆⠐⠽⠾⠷⠒⠭⠘⠆⠐⠽⠾
⠷⠒⠭⠘⠆⠐⠽⠾

14.18.1 *Logical Mathematical Units*— Also divide before any previous item on the priority list.

Example 14.18-2 $(3x^2y)(3x^2y)(3x^2y)(3x^2y)(3x^2y) = 243x^{10}y^5$

⠷⠒⠭⠘⠆⠐⠽⠾⠷⠒⠭⠘⠆⠐⠽⠾⠷⠒⠭⠘⠆⠐⠽⠾⠷⠒⠭⠘⠆⠐⠽⠾
⠷⠒⠭⠘⠆⠐⠽⠾
⠨⠅⠀⠼⠆⠲⠒⠭⠘⠂⠴⠐⠽⠘⠢

If division within an enclosed factor is unavoidable, divide the factor according to the priority list principles. The **challenge** problem at the end of this section illustrates.

14.19 PRIORITY #7—After a Termination Indicator: A division may be made after a termination indicator of a radical expression or after a termination indicator of a modified expression.

Example 14.19-1 $\sqrt[5]{32a^5b^5c^5d^5}\ \sqrt[5]{2abc^2d^3e^2}$

⠣⠢⠜⠒⠆⠁⠘⠢⠐⠃⠘⠢⠐⠉⠘⠢⠐⠙⠘⠢⠐⠻
⠣⠢⠜⠆⠁⠃⠉⠘⠆⠐⠙⠘⠒⠐⠑⠘⠆⠐⠻

Example 14.19-2 $\dfrac{b-a}{n}\sum_{i=0}^{n-1} f\left(a+i\,\frac{b-a}{n}\right) \approx \int_a^b f(x)\,\mathrm{d}x$

Displayed:

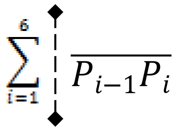
⠀⠀⠹⠃⠤⠁⠌⠝⠼⠐⠨⠠⠎⠩⠊⠀⠨⠅⠀⠼⠴⠣⠝⠤⠂⠻
⠀⠀⠀⠀⠋⠷⠁⠬⠊⠹⠃⠤⠁⠌⠝⠼⠾
⠀⠀⠀⠀⠈⠱⠈⠱⠀⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠙⠭

The five components of a modified expression should not be separated from each other. If, however, the modified expression as a whole is so long that it cannot fit inside the current margins, a division is made in the order of preference listed in this section. [No example given.]

Instructions: Assume this transcription is restricted to a 30-cell line. All of these expressions are longer than 30 cells and require only one division. Draw a vertical line showing where you would divide each expression, keeping in mind the priority sites discussed in this section: before a level indicator, between factors, or after a termination indicator. Then comment on the rule. If you care to transcribe this practice, answers are also given. An example is given first.

Example: $\sum_{i=1}^{6} \overline{P_{i-1}P_i}$

Answer: Divide after the termination indicator of the modified Sigma expression.



Transcription: ⠐⠨⠠⠎⠩⠊⠀⠨⠅⠀⠼⠂⠣⠖⠻
⠀⠀⠐⠠⠏⠰⠊⠤⠂⠐⠠⠏⠰⠊⠐⠣⠱⠻

PRACTICE 14G

1. Multiply these four factors: $(4x^9y)(7x^8y^2)(5x^7y^3)(6x^6y^4)$

2. $\sqrt[3]{x^1}\ \sqrt[6]{x^2}\ \sqrt[12]{x^3}\ \sqrt[24]{x^4}\ \ldots$

3. $N_{x_{y_{z_{\ldots}}}}^{a^{b^{c^{\ldots}}}}$

14.20 Application of this Rule to Special Linked Expressions: Braille format regarding a "special linked expression" is discussed in **Lesson 9**. Here is a review of the layout required in the print copy in order to apply the special formatting in braille:

- The expression is displayed.
- The comparison signs are vertically aligned.
- Other than the anchor on the first line, no sign of comparison is preceded by any expression on its left, with the exception of the last line of the expression which may contain more than one link.

14.20.1 Long Anchor: If the anchor will not fit on one braille line, the guidelines regarding division of mathematical expressions are applied. The runover line to the anchor is indented four cells from the first cell of the anchor.

Example 14.20-1

2. What is the function of the parentheses in the following problem?

$$(4 \times 10{,}000) + (4 \times 1{,}000) + (4 \times 100) + (4 \times 10) + (4 \times 1)$$
$$= 40{,}000 + 4{,}000 + 400 + 40 + 4$$
$$= 44{,}444$$

```
1  ⠼⠃⠲ ⠠⠱⠁⠞ ⠊⠎ ⠮ ⠋⠥⠝⠉⠰⠝ ⠷ ⠮ ⠏⠜⠢⠮⠎⠑⠎ ⠔ ⠮
2    ⠋⠕⠇⠇⠪⠬ ⠏⠗⠕⠃⠇⠑⠍⠦ ⠸⠩
3      ⠷⠲⠈⠡⠂⠴⠠⠴⠴⠴⠾⠬⠷⠲⠈⠡⠂⠠⠴⠴⠴⠾⠬⠷⠲⠈⠡⠂⠴⠴⠾
4          ⠬⠷⠲⠈⠡⠂⠴⠾⠬⠷⠲⠈⠡⠂⠾
5        ⠨⠅ ⠼⠲⠴⠠⠴⠴⠴⠬⠲⠠⠴⠴⠴⠬⠲⠴⠴⠬⠲⠴⠬⠲
6        ⠨⠅ ⠼⠲⠲⠠⠲⠲⠲ ⠸⠱
```

Lines 3-4: This anchor begins in cell 5. The runover begins in cell 9.
Lines 5-6: When the anchor begins in cell 5, each link begins in cell 7.

14.20.2 Long Link: Similarly, if a link will not fit on one braille line, the guidelines regarding division of mathematical expressions are applied to that link. The runover line is indented two cells from the first cell of the link.

Example 14.20-2

We can find the product of 6 and 44,444 by the distributive principle:

$$\begin{aligned} 6 \times 44{,}444 &= 6 \times (40{,}000 + 4{,}000 + 400 + 40 + 4) \\ &= (6 \times 40{,}000) + (6 \times 4{,}000) + (6 \times 400) + (6 \times 40) + (6 \times 4) \\ &= 240{,}000 + 24{,}000 + 2{,}400 + 240 + 24 \\ &= 266{,}664 \end{aligned}$$

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  ⠠⠺⠑ ⠉ ⠋⠔⠙ ⠮ ⠏⠗⠕⠙⠥⠉⠞ ⠷ ⠼⠋ ⠯ ⠼⠙⠙⠂⠙⠙⠙
⠃⠽ ⠮ ⠲⠞⠗⠊⠃⠥⠞⠊⠧⠑ ⠏⠗⠔⠉⠊⠏⠇⠑⠒ ⠸⠩
  ⠼⠖⠈⠡⠲⠲⠠⠲⠲⠲
    ⠨⠅ ⠼⠖⠈⠡⠷⠲⠴⠠⠴⠴⠴⠬⠲⠠⠴⠴⠴⠬⠲⠴⠴⠬⠲⠴⠬⠲⠾
    ⠨⠅ ⠷⠖⠈⠡⠲⠴⠠⠴⠴⠴⠾⠬⠷⠖⠈⠡⠲⠠⠴⠴⠴⠾
      ⠬⠷⠖⠈⠡⠲⠴⠴⠾⠬⠷⠖⠈⠡⠲⠴⠾⠬⠷⠖⠈⠡⠲⠾
    ⠨⠅ ⠼⠆⠲⠴⠠⠴⠴⠴⠬⠆⠲⠠⠴⠴⠴⠬⠆⠠⠲⠴⠴⠬⠆⠲⠴⠬⠆⠲
    ⠨⠅ ⠼⠆⠖⠖⠠⠖⠖⠲ ⠸⠱
```

Line 3: This anchor begins in cell 3.
Lines 4, 5, 7, 8: Each link begins in cell 5.
Line 6: The runover to the second link begins in cell 7.

PRACTICE 14H

The formula derived below is called the *law of cosines.*

$$\begin{aligned} b^2 &= (a\ cos\ B - c)^2 + (a\ sin\ B - 0)^2 \\ &= a^2\ cos^2\ B - 2ac\ cos\ B + c^2 + a^2\ sin^2\ B \\ &= a^2(sin^2\ B + cos^2\ B) - 2ac\ cos\ B + c^2 \\ &= a^2 - 2ac\ cos\ B + c^2 \end{aligned}$$

CHALLENGE PROBLEM #4

By looking at a long expression as a series of logical mathematical units, a clear transcription is possible even if the mathematics is unfamiliar to you. We will analyze the following expression in order to come up with a workable transcription.

$$\frac{1}{2}\left|\int_{\alpha}^{\beta}\left\{x(t)\frac{\mathrm{d}y}{\mathrm{d}t}-y(t)\frac{\mathrm{d}x}{\mathrm{d}t}\right\}\mathrm{d}t\right|$$

There are nested grouping signs: braces within vertical bars.

$$\left|\int_{\alpha}^{\beta}\left\{x(t)\frac{\mathrm{d}y}{\mathrm{d}t}-y(t)\frac{\mathrm{d}x}{\mathrm{d}t}\right\}\mathrm{d}t\right|$$

Material between grouping signs should be kept together on one line, if possible. The material between the braces will fit on one line, so do not divide it.

$$\left\{x(t)\frac{\mathrm{d}y}{\mathrm{d}t}-y(t)\frac{\mathrm{d}x}{\mathrm{d}t}\right\}$$

⠨⠷⠭⠷⠞⠾⠹⠙⠽⠌⠙⠞⠼⠤⠽⠷⠞⠾⠹⠙⠭⠌⠙⠞⠼⠨⠾

The entire grouped expression between the vertical bars will not fit on one line. By dividing before the opening brace, that nested grouping remains undivided.

$$\left|\int_{\alpha}^{\beta}\left\{x(t)\frac{\mathrm{d}y}{\mathrm{d}t}-y(t)\frac{\mathrm{d}x}{\mathrm{d}t}\right\}\mathrm{d}t\right|$$

⠳⠮⠰⠨⠁⠘⠨⠃

⠐⠨⠷⠭⠷⠞⠾⠹⠙⠽⠌⠙⠞⠼⠤⠽⠷⠞⠾⠹⠙⠭⠌⠙⠞⠼⠨⠾⠙⠞⠳

Since a baseline indicator precedes the opening brace, division is made before the baseline indicator according to priority #4.

Here is a clear transcription of the entire expression using "displayed" margins.

$$\frac{1}{2}\left|\int_{\alpha}^{\beta}\left\{x(t)\frac{\mathrm{d}y}{\mathrm{d}t}-y(t)\frac{\mathrm{d}x}{\mathrm{d}t}\right\}\mathrm{d}t\right|$$

⠹⠂⠌⠆⠼⠳⠮⠰⠨⠁⠘⠨⠃

    ⠐⠨⠷⠭⠷⠞⠾⠹⠙⠽⠌⠙⠞⠼⠤⠽⠷⠞⠾⠹⠙⠭⠌⠙⠞⠼⠨⠾⠙⠞⠳

14.21 Margins for Embedded Expressions: When an embedded math expression must be divided, it may begin on the current line provided division is made in accordance with the principles defined above. This is illustrated below using the same expressions in four different layouts.

3-1 narrative format

Example 14.21-1 Is $x = 10{,}000{,}000 + 100{,}000 + 10{,}000 + 1{,}000 + 100 + 10 - 1$ the same as $1 - 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 + 10{,}000{,}000 = x$?

```
  ⠠⠊⠎ ⠸⠩ ⠭
⠨⠅ ⠼⠂⠴⠠⠴⠴⠴⠠⠴⠴⠴⠬⠂⠴⠴⠠⠴⠴⠴⠬⠂⠴⠠⠴⠴⠴⠬⠂⠠⠴⠴⠴
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```

First long expression: Within this narrative format, the anchor "x" fits on line 1. The expression is divided before the equals sign (priority #1) and the link continues on the same line. The long link will not fit on one line so it is divided before an operation sign (priority #2).

Second long expression: Since the long anchor will not fit on one line, it begins on line 3 as it falls within this narrative format. The anchor is divided before an operation sign (priority #2). The last line begins with the equals sign (priority #1) and the short link "x". Note that, although the link will fit on line 4, priority #1 must be observed and so a division is made before the equals sign.

1-3 itemized format

Example 14.21-2

1. Is $x = 10{,}000{,}000 + 100{,}000 + 10{,}000 + 1{,}000 + 100 + 10 - 1$ the same as $1 - 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 + 10{,}000{,}000 = x$?

```
⠼⠁⠲ ⠠⠊⠎ ⠸⠩ ⠭
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  ⠬⠂⠠⠴⠴⠴⠬⠂⠴⠠⠴⠴⠴⠬⠂⠴⠴⠠⠴⠴⠴⠬⠂⠴⠠⠴⠴⠴⠠⠴⠴⠴
  ⠨⠅ ⠭ ⠸⠱⠦
```

Comments are similar to Example 14.21-1, above. Runovers begin in cell 3.

3-5 subitem format

Example 14.21-3

1. Answer each question below.

  a. Is $x = 10{,}000{,}000 + 100{,}000 + 10{,}000 + 1{,}000 + 100 + 10 - 1$ the same as $1 - 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 + 10{,}000{,}000 = x$?

```
⠼⠁⠲ ⠠⠁⠝⠎⠺⠻ ⠑⠁⠡ ⠐⠟ ⠆⠇⠲
  ⠁⠲ ⠠⠊⠎ ⠸⠩ ⠭
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    ⠬⠂⠠⠴⠴⠴⠬⠂⠴⠴⠬⠂⠴⠤⠂ ⠸⠱ ⠮ ⠎⠁⠍⠑ ⠵
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```

Comments are similar to Example 14.21-1, above. Runovers begin in cell 5, restricting the number of cells available on each runover line. Although the short link "x" will fit on line 6, priority #1 must be observed and so a division is made before the final equals sign.

5-3 instructions format

Example 14.21-4

Use this statement to answer each question below: $x = 10{,}000{,}000 + 100{,}000 + 10{,}000 + 1{,}000 + 100 + 10 - 1$

1. …

```
    ⠠⠥⠎⠑ ⠹ ⠌⠁⠞⠑⠰⠞ ⠞⠕ ⠁⠝⠎⠺⠻ ⠑⠁⠡ ⠐⠟ ⠆⠇⠒
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```

The embedded expression begins in the runover cell (cell 3) since the first logical unit (the anchor) does not fit on the first line of the instructions. Division is then made according to the priority list – first before the equals sign (priority #1) and also within the link, before an operation sign (priority #2).

The BANA Nemeth Code Technical Committee is discussing details regarding division of mathematical expressions between braille pages. This section will be completed after decisions are made.

You may have noticed that the current edition of the Nemeth Code has eight items in the priority list, not seven. The BANA Nemeth Code Technical Committee is discussing the reason behind the Nemeth Code's inclusion of "after a comma which occurs between items in an enclosed list" in the priority list rather than as a separate item, as presented in this lesson manual.

14.22 Items That Must Not Be Divided: When a long expression needs to be divided between lines, keep in mind that many symbols and expressions must not be divided.

14.22.1 Symbols to Keep Together: The components of the following symbols must not be divided between braille lines:

— Signs of operation using plus and minus (Lesson 6)
— Signs of comparison compounded vertically or horizontally (Lesson 6)
— Signs of shape with structural or interior modification (Lesson 11)
— Superposed signs (Lesson 13)
— Tally marks belonging to the same group ⠸⠸⠸⠸⠸ (Lesson 13)

14.22.2 Expressions to Keep Together: The following expressions must not be divided between braille lines, even if divided in print:

— A hyphenated expression containing one or more mathematical components (Lesson 3)
— An abbreviation and its related numeral or letter (Lessons 4 and 5)
— A sign of shape and its identifying numeral, letter, or sequence of letters (Lesson 11)
— Characters within a keystroke construction (Lesson 11)
— The components of an expression modified according to the 5-step rule (Lesson 12)
— A function name and the sign following it (Lesson 14)

Spatial Arrangements, continued

SQUARE ROOT DIVISION

14.23 Review of Terminology: Radical expressions were presented in **Lesson 9**. When an answer is shown, a spatial arrangement is required. Here are the names of the parts of a radical expression.

12	*root*	The line above the radicand is the *vinculum*.
√ 144	*radicand*	√ is the *radical sign.*

14.24 Spatial Arrangement for Square Root Problems: In the spatially-arranged radical expression, the first cell of the vinculum is brailled directly above the radical symbol. The last cell of the vinculum extends one cell beyond the radicand. A termination indicator is not required in a spatial square root arrangement.

Radical Sign (with Vinculum)	$\sqrt{}$	⠒⠒⠒ ⠜

⫸ $$\begin{array}{r} 12 \\ \sqrt{144} \end{array}$$

```
  ⠂⠆
⠒⠒⠒⠒⠒
⠜⠂⠲⠲
```

A blank line precedes and follows a spatial arrangement.

Example 14.24-1 12 × 12 = 144 so the square root of 144 is 12, written as follows.

$$\begin{array}{r} 12 \\ \sqrt{144} \end{array}$$

```
  ⠸⠩ ⠼⠂⠆⠈⠡⠂⠆ ⠨⠅ ⠼⠂⠲⠲ ⠸⠱ ⠎ ⠮ ⠎⠟⠥⠜⠑ ⠗⠕⠕⠞
⠷ ⠼⠁⠙⠙ ⠊⠎ ⠼⠁⠃⠂ ⠺⠗⠊⠞⠞⠢ ⠵ ⠋⠕⠇⠇⠪⠎⠲ ⠸⠩
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
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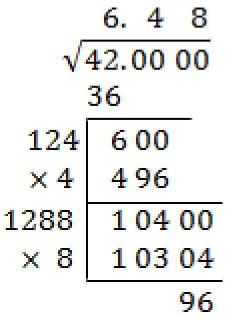
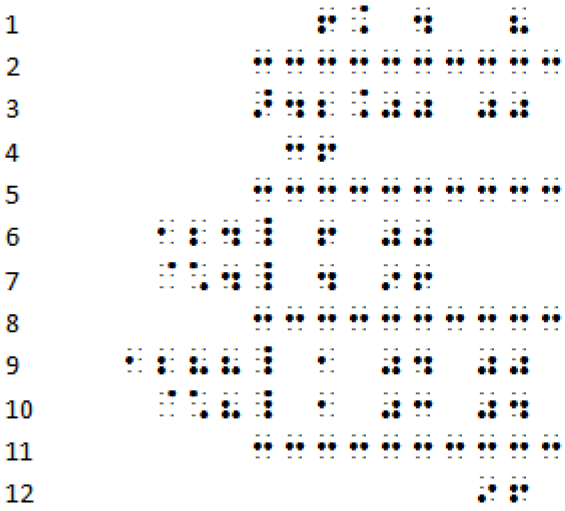
⠸⠱
```

The procedures used with long division arrangements are applied to a spatially-arranged square root problem. (Review alignment and spacing rules for long division in **Lesson 13**.) The vertical line that separates the parts of the problem is represented by dots 456. Spacing between digits replicates spacing in the print copy.

Example 14.24-2

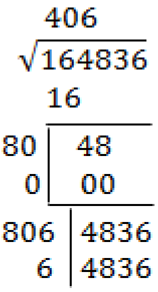


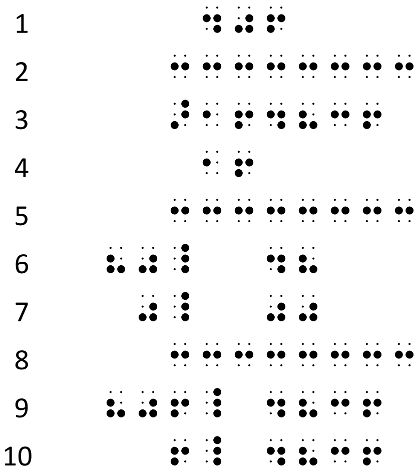


All lines: Spacing between digits matches print in order to attain proper vertical alignment.
Line 2: The vinculum begins in the cell above the radical sign and ends one cell beyond the rightmost character in the entire arrangement.
Lines 5, 8, 11: Separation lines are all the same width.
Lines 6, 7, 9, 10: These vertical lines align below the radical sign.
Lines 7, 10: The multiplication cross is unspaced from the multiplier.

Follow print regarding the alignment of indented vertical lines.

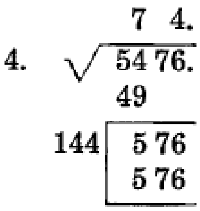
Example 14.24-3

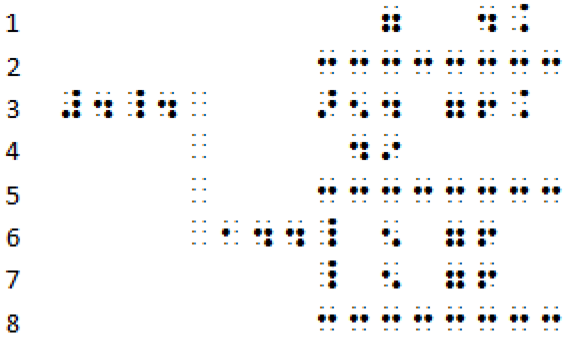




14.25 Placement of Identifiers with Spatial Radical Expressions: An identifier, if present, is placed on the line with the radicand. One blank space is left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

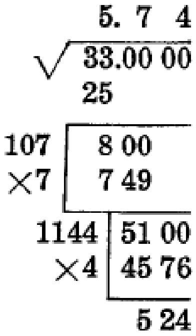
Example 14.25-1





PRACTICE 14I

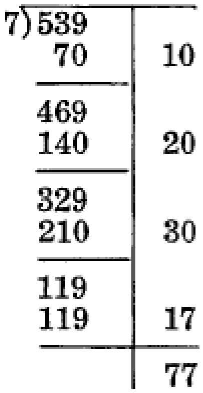
(A)

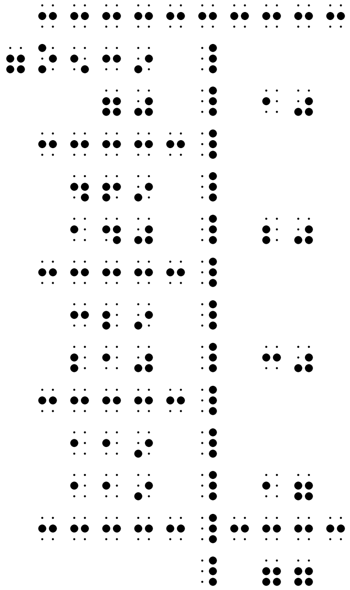


OTHER PRINT LAYOUTS SHOWING DIVISION

14.26 Partial Quotients: This layout shows partial quotients printed to the right of the division problem. A vertical line separates the partial quotients from the rest of the problem. The vertical line may be either drawn as a tactile graphic or it may be represented by dots 456. One space (one blank cell) is left between the vertical line and any digit preceding or following it.

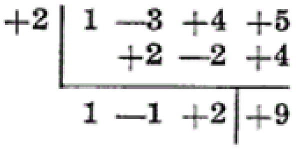
Example 14.26-1



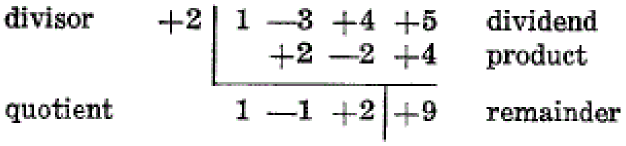


Notice the comparative lengths of the separation lines as well as their vertical alignment.

14.27 Synthetic Division: Synthetic division is a method of showing division of polynomials. There is not a standard print layout. The braille transcription replicates the print design, following alignment rules discussed below, and using the standard separation line and vertical line of the Nemeth Code. Here is an example of one possible layout of a synthetic division problem.



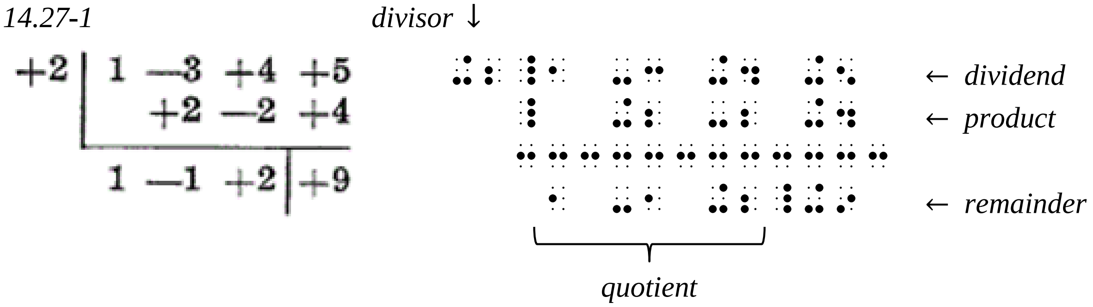
The parts to this problem are labeled as follows.



14.27.1 Alignment and Spacing: In the examples which follow, look carefully at the vertical alignment. The numerals in the dividend, product, and quotient are aligned in vertical columns as in the print copy. Signs of operation, if any, are also vertically aligned. At least one blank cell is left between adjacent columns.

14.27.2 Vertical Line: Braille dots 456 represents the vertical line in the print copy. The braille symbol is shown between the divisor and the division arrangement, beginning on the line with the dividend and ending on the line with the product. No space is left between the vertical line and the dividend or divisor. The separation line (dots 25) extends from the vertical line to one cell beyond the entire arrangement. Another unspaced vertical line is brailled between the quotient and the remainder.

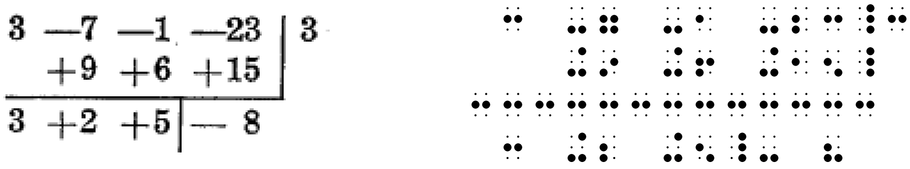
Example 14.27-1



Note the vertical alignment of the numerals and the operation signs.

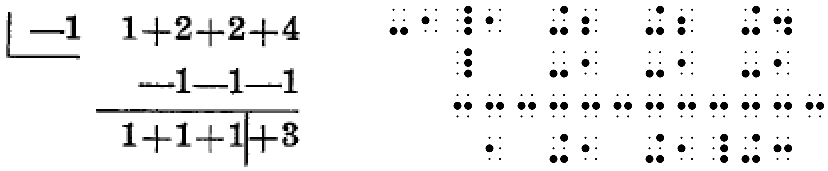
14.27.3 Another Print Style—Divisor on the Right: If the divisor is printed to the right of the overall problem, the same layout is followed in braille. Follow the alignment and spacing rules outlined above, particularly noting that at least one blank cell must be left between adjacent columns. The vertical lines are unspaced from the dividend and the divisor, as well as from the quotient and the remainder.

Example 14.27-2

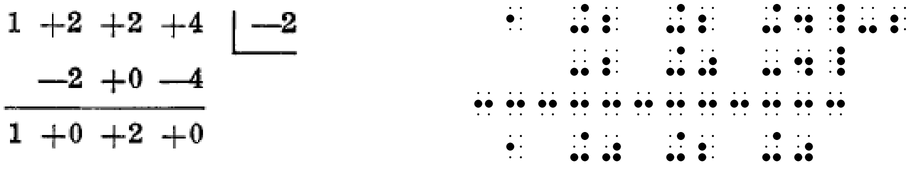


14.27.4 Another Print Style—Boxed Divisor: If the divisor appears boxed in on two sides, the boxing is omitted. A vertical line between the divisor and the dividend is brailled in order to differentiate the divisor from the rest of the arrangement, even though this vertical line does not appear in print. Follow the same alignment and spacing rules outlined above. The first example shows the divisor at the left; the second shows the divisor at the right.

Example 14.27-3



Example 14.27-4



Note that this example has no remainder.

14.27.5 Placement of Identifiers with Synthetic Division: An identifier, if present, is placed on the line with the dividend. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 14.27-5 **197.**

$$\begin{array}{r|rrrr} +2 & 1 & +6 & -1 & -30 \\ \hline & & +2 & +16 & +30 \\ \cline{2-5} \end{array}$$

```
⠼⠂⠔⠶⠸⠲ ⠬⠆⠸⠂ ⠬⠖ ⠤ ⠂ ⠤⠒⠴      ← dividend
         ⠸  ⠬⠆ ⠬⠂⠖ ⠬⠒⠴
         ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
```

Notice the vertical alignment of the operation signs. The numerals are aligned by place value, with the "1" directly above the "6" of "16".

Instructions: In the partial quotients problem, align the minus signs with the division symbol.

PRACTICE 14J

$$\begin{array}{rrrr|l} & 1 & 3 & 2 & \\ 6 \,\big) & 7 & 9 & 2 & \\ - & \underline{6} & \underline{0} & \underline{0} & 1\;0\;0 \\ & 1 & 9 & 2 & \\ - & & 6 & 0 & 1\;0 \\ \hline & 1 & 3 & 2 & \\ & - & \underline{6} & \underline{0} & 1\;0 \\ & & 7 & 2 & \\ & - & \underline{6} & \underline{0} & 1\;0 \\ & & 1 & 2 & \\ & - & \underline{1} & \underline{2} & 2 \\ & & & 0 & \end{array}$$

Dividing Polynomials: Divide $(3x^4 + 12x^3 - 5x^2 - 18x + 8) \div (x + 4)$

$$\begin{array}{r|rrrrr} -4 & 3 & 12 & -5 & -18 & 8 \\ \hline & & -12 & 0 & 20 & -8 \\ \cline{2-6} & 3 & 0 & -5 & 2 & \multicolumn{1}{|r}{0} \end{array}$$

Answer: $3x^2 - 5x - 2$

For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL

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⠷⠶⠾ ⠹⠂⠌⠆⠼⠇⠝ ⠳⠎⠑⠉ ⠼⠆⠞⠬⠞⠁⠝ ⠼⠇⠞⠳⠬⠠⠉
⠷⠦⠾ ⠁⠎⠊⠝ ⠹⠤⠌⠁⠼⠡⠹⠂⠌⠁⠼ ⠨⠅ ⠎⠊⠝ ⠹⠭⠌⠁⠼ ⠸⠱

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⠉⠕⠎ ⠼⠆⠆⠢⠘⠨⠡ ⠨⠅ ⠤⠜⠹⠂⠬⠉⠕⠎ ⠼⠲⠢⠴⠘⠨⠡⠐⠌⠆⠼⠻
⠷⠠⠋⠾ ⠠⠁⠗⠉⠠⠞⠁⠝ ⠈⠷⠭⠠ ⠽⠈⠾ ⠸⠱ ⠛⠊⠧⠑⠎ ⠮
⠜⠉ ⠞⠁⠝⠛⠢⠞ ⠷ ⠸⠩ ⠹⠽⠌⠭⠼ ⠸⠱⠂ ⠞⠁⠅⠬ ⠔⠞⠕ ⠁⠒⠨⠞
⠱ ⠏⠥⠁⠙⠗⠁⠝⠞ ⠮ ⠏⠕⠔⠞ ⠸⠩ ⠷⠭⠠ ⠽⠾ ⠸⠱ ⠊⠎ ⠊⠝⠲
⠐⠣⠰⠠⠛⠐⠜ ⠠⠮ ⠜⠉ ⠞⠁⠝⠛⠢⠞ ⠷ ⠮ ⠉⠕⠍⠏⠇⠑⠭ ⠝⠥⠍⠃⠻
⠰⠏ ⠊⠎ ⠺⠗⠊⠞⠞⠢ ⠸⠩ ⠦⠠⠁⠗⠉⠠⠞⠁⠝ ⠈⠷⠏⠈⠾⠸⠴ ⠸⠱⠲
⠐⠣⠰⠠⠓⠐⠜ ⠠⠒⠎⠊⠙⠻ ⠮ ⠕⠗⠙⠻⠫ ⠏⠁⠊⠗
⠸⠩ ⠷⠉⠕⠎ ⠼⠨⠦⠴⠴⠴⠠ ⠆⠉⠕⠎ ⠼⠨⠦⠴⠴⠴⠾ ⠸⠱⠲

⠠⠠⠏⠗⠁⠉⠞⠊⠉⠑ ⠼⠁⠙⠠⠉

⠸⠩
⠼⠂⠸⠲ ⠇⠕⠛⠰⠝ ⠼⠨⠂⠆⠢ ⠨⠅ ⠤⠼⠨⠖
⠼⠆⠸⠲ ⠁⠝⠞⠊⠇⠕⠛⠰⠁ ⠭ ⠨⠅ ⠠⠝
⠼⠒⠸⠲ ⠇⠕⠛⠨⠴⠢⠲⠒ ⠭ ⠨⠅ ⠤⠼⠨⠶
⠼⠲⠸⠲ ⠉⠕⠞⠘⠤⠂ ⠭⠬⠹⠨⠏⠌⠆⠼⠤⠞⠁⠝⠘⠤⠂ ⠭
⠼⠢⠸⠲ ⠎⠊⠝⠘⠆ ⠼⠔⠴⠘⠨⠡⠐⠬⠉⠕⠎⠘⠆ ⠼⠔⠴⠘⠨⠡ ⠨⠅ ⠼⠂
⠼⠖⠸⠲ ⠑⠘⠭⠬⠇⠝ ⠭
⠼⠶⠸⠲ ⠑⠘⠎⠊⠝ ⠭⠐⠬⠑⠘⠎⠊⠝ ⠽
⠼⠦⠸⠲ ⠼⠆⠘⠎⠑⠉ ⠭ ⠨⠅ ⠽ ⠸⠱

⠠⠠⠏⠗⠁⠉⠞⠊⠉⠑ ⠼⠁⠙⠠⠙

⠼⠁⠲ ⠠⠋⠔⠙ ⠸⠩
⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠼⠴⠨⠖⠻ ⠼⠆⠘⠆⠢⠭⠘⠘⠆⠘⠤⠂⠴⠭⠤⠂ ⠸⠱⠲
⠼⠃⠲ ⠠⠿⠍⠥⠇⠁⠞⠑ ⠁ ⠏⠗⠑⠉⠊⠎⠑ ⠙⠑⠋⠔⠊⠰⠝ ⠿
⠸⠩ ⠐⠇⠊⠍⠩⠭ ⠫⠩⠒⠒⠕ ⠤⠠⠿⠻ ⠋⠷⠭⠾ ⠨⠅ ⠠⠇⠸⠲
⠼⠒⠸⠲ ⠠⠄⠠⠊⠋ ⠐⠣⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠁⠰⠝ ⠨⠅ ⠠⠁ ⠠⠄⠯
⠐⠣⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠃⠰⠝ ⠨⠅ ⠠⠃ ⠸⠱⠂ ⠍⠌ ⠭ ⠆
⠞⠗⠥⠑ ⠞ ⠸⠩
⠐⠣⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠷⠁⠰⠝⠐⠬⠃⠰⠝⠐⠾ ⠨⠅ ⠠⠁⠬⠠⠃⠸⠦
⠼⠲⠸⠲ ⠠⠄⠠⠋⠔⠙ ⠐⠣⠇⠊⠍⠊⠞⠩⠝ ⠫⠕ ⠠⠿⠻ ⠁⠰⠝ ⠠⠄⠱⠢
⠁⠰⠝ ⠨⠅ ⠷⠤⠂⠾⠰⠝⠸⠲
⠼⠢⠸⠲ ⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠼⠴⠻ ⠉⠎⠉ ⠭⠇⠝ ⠷⠂⠬⠭⠾ ⠸⠱

⠠⠠⠏⠗⠁⠉⠞⠊⠉⠑ ⠼⠁⠙⠠⠑

⠸⠩

⠰⠠⠁⠸⠲ ⠼⠆⠈⠡⠲⠆⠒ ⠨⠅ ⠷⠆⠈⠡⠲⠴⠴⠾⠬⠷⠆⠈⠡⠆⠴⠾⠬⠷⠆⠈⠡⠒⠾
⠀⠀⠨⠅ ⠼⠦⠴⠴⠬⠲⠴⠬⠖ ⠨⠅ ⠼⠦⠲⠖
⠰⠠⠃⠸⠲ ⠫⠪ ⠼⠂⠬⠫⠪ ⠼⠆⠬⠫⠪ ⠼⠒⠬⠫⠪ ⠼⠲⠬⠫⠪ ⠼⠢
⠀⠀⠬⠫⠪ ⠼⠖⠬⠫⠪ ⠼⠶
⠀⠀⠨⠅ ⠼⠲⠔⠴⠘⠨⠡
⠰⠠⠉⠸⠲
⠀⠀⠼⠆⠸⠹⠒⠌⠲⠸⠼ ⠽⠙ ⠬⠂⠸⠹⠒⠌⠲⠸⠼ ⠽⠙ ⠬⠹⠒⠌⠲⠼ ⠽⠙
⠀⠀⠨⠅ ⠼⠢⠸⠹⠂⠌⠲⠸⠼ ⠽⠙
⠰⠠⠙⠸⠲ ⠜⠷⠭⠬⠁⠘⠆⠐⠾⠬⠷⠽⠬⠁⠘⠆⠐⠾⠻
⠀⠀⠤⠜⠷⠭⠤⠁⠘⠆⠐⠾⠤⠷⠽⠤⠁⠘⠆⠐⠾⠻
⠀⠀⠨⠅ ⠬⠤⠆⠁ ⠸⠱

⠀⠀⠘⠂⠠⠎⠕⠍⠍⠻⠄⠎ ⠘⠂⠠⠗⠳⠞⠔⠑⠒ ⠠⠎⠕⠍⠍⠻⠄⠎ ⠗⠳⠞⠔⠑ ⠉
⠆ ⠗⠑⠏⠗⠑⠎⠢⠞⠫ ⠃⠽ ⠮ ⠋⠕⠇⠇⠪⠬ ⠑⠭⠏⠗⠑⠎⠨⠝⠲
⠀⠀⠠⠗⠳⠞⠔⠑ ⠰⠠⠎⠒ ⠸⠩ ⠼⠢⠬⠒⠬⠲⠷⠂⠬⠐⠤⠂⠾⠬⠷⠤⠒⠾⠬⠷⠤⠢⠾
⠀⠀⠀⠀⠬⠒⠷⠢⠬⠷⠤⠆⠾⠾⠬⠂ ⠸⠱
⠠⠙⠗⠁⠺ ⠁ ⠎⠊⠍⠏⠇⠑ ⠙⠊⠁⠛⠗⠁⠍ ⠞⠕ ⠗⠑⠏⠗⠑⠎⠢⠞
⠨⠂⠠⠗⠳⠞⠔⠑ ⠨⠆⠰⠠⠎⠲

B and D: Since a division must be made before an operation sign (priority #2), a division must also be made before the equals sign (priority #1).

C: The anchor will fit on one line if it starts in the runover position of this itemized problem (cell 3). The only necessary division is made before the equals sign (priority #1).

⠠⠠⠏⠗⠁⠉⠞⠊⠉⠑ ⠼⠁⠙⠠⠋

⠠⠉⠕⠍⠏⠥⠞⠑ ⠯⠸⠌⠕⠗ ⠎⠊⠍⠏⠇⠊⠋⠽⠲ ⠸⠩

⠼⠂⠸⠲ ⠹⠷⠂⠨⠆⠂⠈⠡⠂⠴⠘⠲⠐⠾⠷⠖⠨⠔⠒⠶⠈⠡⠂⠴⠘⠦⠐⠾
⠌⠒⠨⠶⠢⠷⠂⠴⠘⠆⠐⠾⠼

⠼⠆⠸⠲ ⠠⠹⠷⠹⠒⠌⠆⠼⠾⠈⠡⠷⠹⠂⠌⠆⠼⠾⠈⠡⠷⠤⠹⠂⠌⠆⠼⠾
⠠⠌⠂⠈⠡⠆⠈⠡⠒⠠⠼

⠼⠒⠸⠲ ⠷⠬⠤⠾⠁⠰⠂⠊⠰⠰⠂⠐⠁⠰⠆⠊⠰⠰⠆⠐⠁⠰⠒⠊⠰⠰⠒⠐⠁⠰⠲⠊⠰⠰⠲
⠐⠄⠄⠄ ⠁⠰⠝⠊⠰⠰⠝

⠼⠲⠸⠲ ⠷⠂⠖⠭⠘⠲⠐⠬⠦⠭⠘⠒⠐⠽⠬⠲⠭⠘⠆⠐⠽⠘⠆⠐⠬⠆⠭⠽⠘⠒
⠐⠬⠽⠘⠲⠐⠾⠷⠆⠭⠤⠽⠾
⠨⠅ ⠼⠒⠆⠭⠘⠢⠐⠤⠽⠘⠢

⠼⠢⠸⠲ ⠼⠂⠲⠲ ⠋⠞⠘⠆⠐⠬⠂⠴⠶ ⠋⠞⠘⠆⠐⠬⠂⠂⠆ ⠋⠞⠘⠆
⠐⠬⠂⠲⠢ ⠋⠞⠘⠆⠐⠤⠂⠒⠂ ⠋⠞⠘⠆
⠨⠅ ⠠⠝

⠼⠖⠸⠲ ⠹⠜⠴⠨⠲⠢⠆⠒⠻⠈⠡⠷⠆⠨⠖⠆⠶⠢⠾⠘⠒
⠐⠌⠲⠒⠖⠔⠴⠈⠡⠴⠨⠴⠴⠴⠔⠦⠆⠼

⠼⠶⠸⠲
⠬⠤⠷⠹⠁⠘⠹⠆⠌⠒⠼⠐⠬⠁⠘⠹⠂⠌⠒⠼⠐⠃⠘⠹⠂⠌⠒⠼⠐⠬⠃⠘⠹⠆⠌⠒⠼
⠐⠌⠒⠼⠾⠘⠹⠒⠌⠆⠼

⠼⠦⠸⠲ ⠠⠹⠂⠨⠶⠔⠒ ⠰⠛ ⠠⠉⠥⠸⠌⠰⠛ ⠠⠉⠇
⠠⠌⠴⠨⠦⠔⠖⠲ ⠰⠛ ⠠⠉⠥⠸⠌⠰⠛ ⠠⠉⠇⠠⠼
⠨⠅ ⠤⠤⠤⠤ ⠸⠱

In #8, although the link will fit on the previous line, because a division was made at the fraction line (priority #3), a division must also be made before the comparison sign (priority #1).

PRACTICE 14G

1. Multiply these four factors: $(4x^9y)(7x^8y^2)(5x^7y^3)(6x^6y^4)$

Divide between factors.

⠼⠁⠲ ⠠⠍⠥⠇⠞⠊⠏⠇⠽ ⠘⠮ ⠋⠳⠗ ⠋⠁⠉⠞⠕⠗⠎⠒
⠸⠩ ⠷⠲⠭⠘⠔⠐⠽⠾⠷⠶⠭⠘⠦⠐⠽⠘⠆⠐⠾
⠷⠢⠭⠘⠶⠐⠽⠘⠒⠐⠾⠷⠖⠭⠘⠖⠐⠽⠘⠲⠐⠾

2. $\sqrt[3]{x^1}\ \sqrt[6]{x^2}\ \sqrt[12]{x^3}\ \sqrt[24]{x^4}\ \dots$

Divide after the termination indicator of a radical expression.

⠼⠆⠸⠲ ⠣⠒⠜⠭⠘⠂⠐⠻⠣⠖⠜⠭⠘⠆⠐⠻⠣⠂⠆⠜⠭⠘⠒⠐⠻
⠣⠆⠲⠜⠭⠘⠲⠐⠻ ⠄⠄⠄

3. $N_{x_{y_{z_{\dots}}}}^{a^{b^{c^{\cdots}}}}$

Divide before the superscript level indicator.

⠼⠒⠸⠲ ⠠⠝⠰⠭⠰⠰⠽⠰⠰⠰⠵⠰⠰⠰⠰⠄⠄⠄
⠘⠁⠘⠘⠃⠘⠘⠘⠉⠘⠘⠘⠘⠄⠄⠄ ⠸⠱

⠠⠠⠏⠗⠁⠉⠞⠊⠉⠑ ⠼⠁⠙⠠⠛

⠠⠮ ⠿⠍⠥⠇⠁ ⠙⠻⠊⠧⠫ ⠆⠇ ⠊⠎ ⠉⠁⠇⠇⠫ ⠮ ⠨⠶⠇⠁⠺ ⠷
⠉⠕⠎⠔⠑⠎⠲⠨⠄ ⠸⠩
⠃⠘⠆
⠨⠅ ⠷⠁⠉⠕⠎ ⠠⠃⠤⠉⠾⠘⠆⠐⠬⠷⠁⠎⠊⠝ ⠠⠃⠤⠴⠾⠘⠆
⠨⠅ ⠁⠘⠆⠐⠉⠕⠎⠘⠆ ⠠⠃⠤⠆⠁⠉⠉⠕⠎ ⠠⠃⠬⠉⠘⠆
⠐⠬⠁⠘⠆⠐⠎⠊⠝⠘⠆ ⠠⠃
⠨⠅ ⠁⠘⠆⠐⠷⠎⠊⠝⠘⠆ ⠠⠃⠬⠉⠕⠎⠘⠆ ⠠⠃⠾
⠤⠆⠁⠉⠉⠕⠎ ⠠⠃⠬⠉⠘⠆
⠨⠅ ⠁⠘⠆⠐⠤⠆⠁⠉⠉⠕⠎ ⠠⠃⠬⠉⠘⠆ ⠸⠱

⠠⠠⠏⠗⠁⠉⠞⠊⠉⠑ ⠼⠁⠙⠠⠊

⠸⠩

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          ⠢⠨ ⠶  ⠲
        ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠷⠠⠁⠾    ⠜⠒⠒⠨⠴⠴ ⠴⠴
         ⠆⠢
        ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
     ⠂⠴⠶⠸ ⠦ ⠴⠴
     ⠈⠡⠶⠸ ⠶ ⠲⠔
        ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
      ⠂⠂⠲⠲⠸ ⠢⠂ ⠴⠴
       ⠈⠡⠲⠸ ⠲⠢ ⠶⠖
          ⠒⠒⠒⠒⠒⠒⠒⠒
             ⠢ ⠆⠲
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⠸⠱

⠠⠠⠏⠗⠁⠉⠞⠊⠉⠑ ⠼⠁⠙⠠⠚

⠸⠩

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  ⠂⠒⠆
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 ⠤⠖⠴⠴ ⠸ ⠂⠴⠴
 ⠒⠒⠒⠒⠒⠸
  ⠂⠔⠆ ⠸
 ⠤ ⠖⠴ ⠸ ⠂⠴
 ⠒⠒⠒⠒⠒⠸
  ⠂⠒⠆ ⠸
 ⠤ ⠖⠴ ⠸ ⠂⠴
 ⠒⠒⠒⠒⠒⠸
   ⠶⠆ ⠸
 ⠤ ⠖⠴ ⠸ ⠂⠴
 ⠒⠒⠒⠒⠒⠸
   ⠂⠆ ⠸
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 ⠒⠒⠒⠒⠒⠸
    ⠴ ⠸
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⠸⠱

⠘⠂⠠⠙⠊⠧⠊⠙⠬ ⠘⠂⠠⠏⠕⠇⠽⠝⠕⠍⠊⠁⠇⠎⠒ ⠠⠙⠊⠧⠊⠙⠑ ⠸⠩ ⠷⠒⠭⠘⠲⠐⠬⠂⠆⠭⠘⠒⠐⠤⠢⠭⠘⠆⠐⠤⠂⠦⠭⠬⠦⠮⠨⠌⠷⠭⠬⠲⠾

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  ⠤⠲⠸⠒  ⠂⠆ ⠤⠢ ⠤⠂⠦  ⠦
    ⠸  ⠤⠂⠆  ⠴  ⠆⠴ ⠤⠦
    ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
     ⠒   ⠴ ⠤⠢   ⠆⠸ ⠴
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⠠⠄⠨⠂⠠⠁⠝⠎⠺⠻⠒ ⠼⠒⠭⠘⠆⠐⠤⠢⠭⠤⠆ ⠸⠱

EXERCISE 14

Exercise 14 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 15.