LESSON 13

**Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.**

MISCELLANEOUS SYMBOLS

In this lesson, symbols are grouped according to spacing rules: unspaced, spaced, and those with special rules. Within each section, the symbols are presented in alphabetical order.

Unspaced Miscellaneous Symbols

13.1 Spacing Rules for Unspaced Symbols: No space is left between the symbols listed below and any other symbol or quantity to which they apply, regardless of print spacing idiosyncrasies. *However*, a space must be left between these symbols and a word, an abbreviation, a sign of comparison, or other symbol which specifically requires a space before or after it.

13.1.1 Caret

Caret	∧	⠘⠪

In addition to the caret being used as a modifier above or below a mathematical expression as shown in **Lesson 12,** it may be used as a place indicator with the decimal point to show the position to which the decimal point has been moved. Used as a place indicator, it is treated as a numeric symbol.

≫   .37ˏ688

Example 13.1-1 The caret (^) shows the place to which the decimal is moved: .37ˏ688

13.1.1.a Use of the UEB Caret: The caret may be brailled more than one way within a document. The distinction is based on meaning. In a technical context, use the Nemeth Code caret. In nontechnical context, use the UEB caret ⠘⠢ to indicate the insertion of literary material, or the UEB circumflex ⠘⠔ to indicate a modified letter or accent. In UEB context, follow UEB rules for spacing of the caret or circumflex symbol.

13.1.2 Crossed Letters

Crossed d	đ	⠈⠙
Crossed h	ħ	⠈⠓
Crossed lambda	ƛ	⠈⠮⠣
Crossed R	℞	⠈�

> 2đv

> $2\pi\hbar v = \phi e$

> $3\lambda \ldots n\lambda$

> $n = 3℞$

Since the "crossed d" is the same symbol as the UEB transcriber-defined shape indicator (often used to define an icon), if the UEB symbol is also used in the volume, both symbols should be listed on the Special Symbols page. Here is a sample.

13.1.3 Del

+---+
| **Del (nabla, gradient)** ∇ ⠨⠦ |
+---+

When the inverted triangle is used as a sign of omission, it is spaced according to the rules presented in **11.27**. In all other cases it is spaced in accordance with **13.1**.

⧐ $s\nabla t + t\nabla s$

⧐ $\nabla(r^n)$

Example 13.1-2   The symbol "∇" is called "del" as in $\mathbf{T} \cdot \nabla (r_1 + r_2) = 0$.

13.1.4 Derivative and Partial Derivative (round d): Derivatives are commonly notated as dx and dy. (You may also encounter dt, du, dv, dz, etc.) Consider dx and dy as symbols meaning "the derivative of x" and "the derivative of y". In print it is quite common that dx and dy are spaced within a mathematical expression. <u>In braille, the space is omitted</u>.

⧐ $(1 + 4xy)\,dx\,dy$

The partial derivative is represented by the following symbol.

+---+
| **Partial Derivative** ∂ ⠙ |
+---+
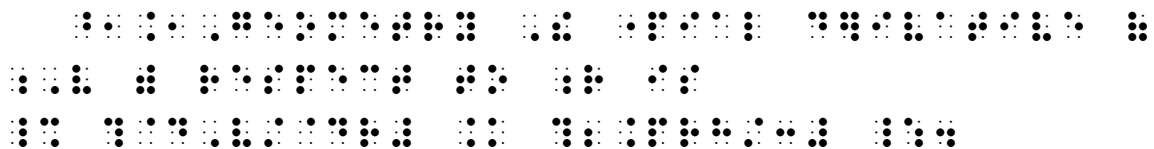
⧐ $\dfrac{\partial f}{\partial x}$

⧐ $\partial/\partial y(\partial b/\partial x)$

⧐ $\dfrac{\partial h}{\partial u} = \dfrac{\partial f}{\partial x}\,\dfrac{\partial x}{\partial u}$

Example 13.1-3   ***Geometry***   The partial derivative of *V* with respect to *r* is $\dfrac{\partial V}{\partial r} = \dfrac{2\pi r h}{3}$.

13.1.5 Empty Set (null set, void set)

Empty Set (null set, void set)		
Represented by Zero With a Slanted or Vertical Bar Through It	$\emptyset$ or Φ	⠰⠴ ⠰⠐⠴
Represented by Facing Braces	{ }	⠐⠷ ⠾ ⠐⠷⠀⠾

The print symbols $\emptyset$ and Φ used to denote the empty set must not be mistaken for the Greek uncapitalized phi (φ or ϕ) which they resemble.

≫ $A \cap \emptyset = \emptyset$

≫ $\emptyset \subseteq A$

When facing braces are used to denote the empty set, one space is left between the braces.

≫ $S \cup \{ \}$

Example 13.1-4   The solution set $\emptyset$ is written $R = \emptyset$ or $R = \{ \}$.

13.1.6 Factorial Sign

Factorial Sign	!	⠨⠮

≫ $n!$

≫ $_5C_1 = \dfrac{5!}{1!4!}\, n!$

Example 13.1-5

Poisson Probability   People enter a line for the *Jack Rabbit Coaster* at the rate of 4 per minute. The following formula can be used to determine the probability that *x* people will arrive within the next minute.

$$P(x) = \frac{4^x e^{-4}}{x!}$$

where

$$x! = x \cdot (x-1) \cdot (x-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1.$$

Determine the probability that $x = 5$ people will arrive within the next minute.

1 ⠀⠀⠀⠠⠏⠕⠊⠎⠎⠕⠝⠀⠠⠏⠗⠕⠃⠁⠃⠊⠇⠊⠞⠽

2 ⠀⠀⠠⠏⠀⠷⠙⠮⠀⠒⠀⠏⠟⠕⠏⠇⠑⠀⠢⠞⠻⠀⠁⠀⠇⠔⠑

3 ⠠⠍⠕⠗⠮⠞⠑⠀⠞⠓⠀⠁⠌⠝⠑⠀⠋⠷⠀⠋⠁⠞⠞⠑⠁⠗⠀⠠⠁

4 ⠶⠕⠇⠇⠁⠂⠀⠼⠙⠂⠅⠕⠈⠀⠠⠀⠉⠀⠛⠻⠀⠼⠑⠀⠍⠊⠝⠥⠞⠑⠂⠀⠉

5 ⠋⠕⠇⠇⠪⠊⠝⠛⠀⠋⠕⠗⠍⠥⠇⠁⠀⠉⠁⠝⠀⠃⠑⠀⠥⠎⠑⠙⠀⠞⠕⠀⠙⠑⠞⠻⠍⠔⠑

6 ⠞⠓⠑⠀⠏⠗⠕⠃⠁⠃⠊⠇⠊⠞⠽⠀⠹⠁⠞⠀⠭⠀⠏⠑⠕⠏⠇⠑⠀⠺⠊⠇⠇

7 ⠀⠀⠀⠠⠏⠷⠭⠾⠀⠨⠅⠀⠹⠆⠬⠲⠬⠢⠬⠲⠬⠬⠄⠨�

8 ⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

9 ⠀⠀⠀⠭⠖⠀⠭⠀⠐⠦⠣���⠀⠱⠖⠄⠐⠣

10 ⠀⠀⠠⠙⠑⠞⠻⠍⠔⠑⠀⠋⠕⠗⠀⠏⠗⠕⠃⠁⠃⠊⠇⠊⠞⠽⠀⠼⠑⠀⠹⠁⠞

11 ⠠⠀⠭⠀⠼⠑⠀⠏⠑⠕⠏⠇⠑⠀⠺⠊⠇⠇⠀⠁⠗⠗⠊⠧⠑

<u>Format Notes</u>

Line 1: The paragraph heading is brailled as a cell-5 heading. Font style (boldface) is disregarded in a cell-5 heading.

Line 2: The paragraph begins in cell 3, following the rules of the Nemeth Code.

Lines 7 and 9: Each displayed math expression begins in cell 3.

Lines 8 and 10: The narrative continues in the runover cell for paragraphs, cell 1.

Note: The last sentence can be interpreted either as a continuation of the same paragraph or as a new paragraph in which case it would begin in cell 3.

13.1.7 Infinity

Infinity	∞	⠒⠿

⧐ $a - (+\infty) = -\infty$ ⠰⠁⠀⠤⠰⠇⠰⠨⠼⠿⠀⠐⠶⠀⠤⠰⠼⠿

⧐ $n \to \infty$ ⠝⠀⠳⠕⠀⠰⠼⠿

⧐ 1^∞ ⠼⠁⠬⠰⠼⠿

$$\sum_{n=-\infty}^{\infty} f(n)$$

⧐   ⠠⠎⠰⠎⠐⠏⠶⠤⠰⠼⠿⠀⠘⠕⠰⠼⠿⠀⠋⠷⠝⠾

Example 13.1-6

	Interval	Inequality
	(a, +∞)	x > a
	[a, +∞)	x ≥ a
	(−∞, a)	x < a
	(−∞, a]	x ≤ a

⠰⠽⠒⠹⠦⠙⠞� ⠰⠇⠀⠀⠀⠰⠽⠒⠹⠒⠹⠰⠇⠀⠞⠀⠰⠎

⠐⠶⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠀⠀⠐⠶⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒

⠰⠷

⠦⠁⠠⠀⠤⠰⠼⠿⠀⠀⠀⠭⠀⠐⠕⠀⠁

⠈⠼⠁⠠⠠⠀⠤⠰⠼⠿⠀⠀⠀⠭⠀⠐⠢⠈⠀⠁

⠦⠤⠰⠼⠿⠠⠀⠞⠁⠀⠀⠀⠭⠀⠪⠀⠁

⠈⠼⠤⠰⠼⠿⠠⠀⠞⠞⠁⠀⠀⠀⠭⠀⠪⠈⠀⠁

⠰⠾

13.1.8 Integral

Single Integral	$\displaystyle\int$	⠮
Double Integral	$\displaystyle\iint$	⠮⠮
Triple Integral	$\displaystyle\iiint$	⠮⠮⠮

≫ $\int (du + dv) = \int du + \int dv$ ⠮⠷⠙⠥⠬⠙⠧⠾ ⠨⠅ ⠮⠙⠥⠬⠮⠙⠧

≫ $V = \iiint dx\, dy\, dy$ ⠠⠧ ⠨⠅ ⠮⠮⠮⠙⠭⠙⠽⠙⠽

Reminder: There are no spaces brailled before and after the terms of the expression (dx and dy) even though they are spaced in print.

Superscripts and subscripts are printed just to the right of the integral symbol. The subscript is brailled first. A subscript indicator is required for numeric subscripts.

≫ $\displaystyle\int_{1}^{3} \frac{dx}{x^2-2x+5}$

⠮⠰⠼⠂⠘⠒⠀⠹⠙⠭⠌⠭⠘⠆⠐⠤⠆⠭⠬⠢⠼⠾

≫ $\displaystyle\int_{1}^{2}\int_{0}^{x-1} y\, dy\, dx$ ⠮⠰⠼⠂⠘⠆⠮⠰⠼⠴⠘⠭⠤⠂⠐⠽⠙⠽⠙⠭

Modifiers (symbols printed directly above and/or directly below the integral symbol) are transcribed according to the five-step rule for the transcription of modified expressions.

≫ $\displaystyle\int_{0}^{\infty} f(x)\,dx$ ⠐⠮⠰⠼⠴⠘⠼⠿⠐⠫⠋⠷⠭⠾⠙⠭

> The single integral sign is modified directly below with the numeral zero and directly above with an infinity symbol.

≫ $\displaystyle\iint_{Q} f(x, y)\, dy\, dx$ ⠐⠮⠮⠰⠟⠐⠫⠋⠷⠭⠠⠀⠽⠾⠙⠽⠙⠭

> The double integral sign is modified directly below with the letter "Q".

We apologize for the blurry images in this lesson. We hope to provide better graphics in future editions.

$$\iint_R 6xy^2 \, \mathrm{d}A \;=\; \int_1^2 \int_2^4 6xy^2 \, \mathrm{d}x \, \mathrm{d}y$$

$$= \int_1^2 (3x^2 y^2) \Big|_2^4 \, \mathrm{d}y$$

$$= \int_1^2 36y^2 \, \mathrm{d}y$$

$$= 12y^3 \big|_1^2$$

$$= 84$$

Can you explain the format, spacing, and indicators in the transcription?

⠫⠃⠮⠆⠆���524⠎�⠭⠽�⠙�

(braille transcription lines)

13.1.8.a Upper and Lower Integral Signs: A horizontal bar directly over or under the integral sign is not a modifier—the bar is an essential part of the symbol. The symbols for the *upper* and *lower integral signs* are transcribed as follows.

Upper Integral	$\overline{\int}$	⠨⠮ ⠮
Lower Integral	$\underline{\int}$	⠠⠮ ⠮

≫ $\overline{\int}_a^b f(x)\mathrm{d}x = 0$

≫ $\underline{\int}_a^b f(x)\mathrm{d}x = 0$

13.1.9 Quantifiers

Existential Quantifier		
There Exists, for Some	∃ or ∃	⠐⠱
There Exists Uniquely for Exactly One	∃| or ∃|	⠐⠱⠳
Universal Quantifier		
For All, For Each, For Every	∀ or ∀	⠈⠁

> $\exists x,\ x < \dfrac{1}{n}$

> $(\exists_x \in A)$

Example 13.1-7 $\exists\vert_v\vert\ v = -v$ means "there exists exactly one *v* such that *v* equals –*v*."

The second vertical bar is a sign of comparison(meaning "such that" as understood from the narrative) and is spaced accordingly. Review 6.7.16.

> $(\forall_x)$

> $\forall_x \forall_y - \dfrac{y-x}{x+y}$

Example 13.1-8 Assuming the domain of variable *s* is ℝ, is the following statement true?

$$(\forall_s)(s \cdot 0 = 0)$$

A transcriber's note would explain the substitution of the script typeform indicator for the double-struck letter R. See Lesson 7.

Instructions: Except for the headings, use Nemeth Code throughout.

PRACTICE 13A

Unspaced Miscellaneous Symbols

1.2$_\wedge$

$(v > \phi\hbar)$

℞: 24 grams

$\|\triangledown f(\mathbf{a})\|$

$$\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}$$

$\{\,\} \cap \{\,\}$

$$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

$-\infty < x < \infty$

$f'(x) = 0 \text{ or } \infty$

$$\int_a^b f(x)\,dx = F(x)\Big]_a^b$$

$$\int_{x=a}^{x=b} f(t)\,dt$$

$(\exists x)(\exists y)[x + y = 85]$

$\exists!_x$

$\forall_x \in A$

Spaced Miscellaneous Symbols

13.2 Spacing Rules for Spaced Symbols: A space is left *before and after* the symbols listed below. However, no space is left between these symbols and a mark which applies to them such as a sign of grouping, a braille indicator, or a punctuation mark.

13.2.1 "At" Sign in Mathematical Context

At Sign	@	⠈⠙

≫ (@)   ⠦⠈⠙⠴

Example 13.2-1   15 pencils @45¢ = $6.75.

⠼⠁⠑ ⠏⠑⠝⠉⠊⠇⠎ ⠈⠙ ⠼�⠑⠬ ⠀⠨⠅ ⠈⠎⠼⠋⠲⠶⠑

Switching Decision: The problem is not "45¢ = $6.75 " therefore "15 pencils @" is part of the problem and is brailled in Nemeth Code. Spacing: In the print copy there is no space following the "at" symbol. In braille, a space is inserted both before and after this symbol.

13.2.2 Check Mark: Within Nemeth Code material, rather than switching out of Nemeth Code to braille a UEB transcriber-defined symbol, the following Nemeth Code symbol may be used.

Check Mark	✓	⠈⠱

A space is left before and after a single check mark.

≫ ✓ 1 + 2 = 3   ⠈⠱ ⠼⠁⠬⠼⠃ ⠨⠅ ⠼⠉

A sequence of two or more check marks is written unspaced, but the combination as a whole is preceded and followed by a space.

≫ ✓✓ 2 + 2 = 4   ⠈⠱⠈⠱ ⠼⠃⠬⠼⠃ ⠨⠅ ⠼⠙

13.2.3 Ditto Marks: A ditto mark is centered beneath the material to which it applies. It is separated from any expression which precedes or follows it by at least one space. In Nemeth Code, the ditto mark shown below is used.

Ditto Marks	"	⠄�

Example 13.2-2　　　　　1.5 times 10 equals　15.0.
　　　　　　　　　　　　" 　"　20　"　　30.0.
　　　　　　　　　　　　" 　"　30　"　　45.0.

⠿ ⠂⠒

⠼⠂⠲ ⠐⠡ ⠼⠂⠴ ⠀⠐⠶ ⠼⠂⠑⠲⠴
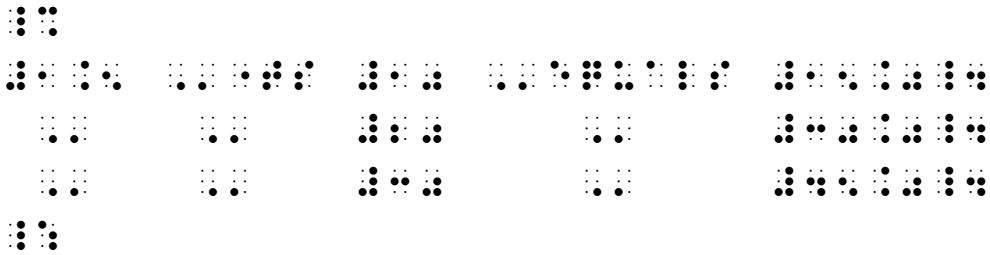
In UEB contexts, the UEB ditto mark symbol is used, but Nemeth Code format is applied: center the ditto under the word or item to which it applies.

Example 13.2-3　　　　　1 times 10 equals　10.
　　　　　　　　　　　　" 　"　20　"　　20.
　　　　　　　　　　　　" 　"　30　"　　30.
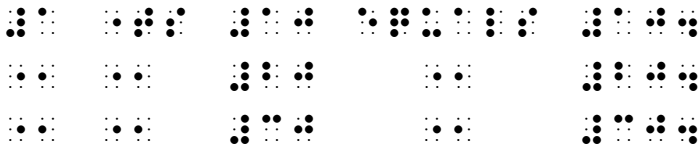
13.2.4 Since (because)

Since (because)	⠡	⠐⠡
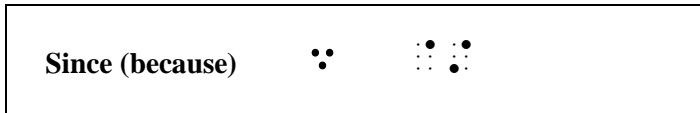
⩾　　$\because x = y, x^2 = y^2$

⩾　　$(\because)$ RS = RT

No space is left between the symbol and grouping signs which apply to it.

13.2.5 Therefore

Therefore		
Normal	∴	⠠⠡
Negated (it does not follow that)	/∴	⠌⠠⠡

≫   ∴ CM ⊥ AB   ⠠⠡ ⠠⠉⠠⠍ ⠀⠯ ⠠⠁⠠⠃

≫   /∴R = S   ⠌⠠⠡ ⠠⠗ ⠶ ⠠⠎

Example 13.2-4   ∴ the solution set is {±3}.

⠠⠡ ⠮ ⠎⠕ ⠇⠥⠰⠹⠝ ⠎⠑⠞ ⠊⠎
⠸⠣⠬⠒⠒⠔⠀⠈⠼⠉⠸⠜

13.2.6 Boldface Vertical Bar (end of proof)

A single boldface vertical bar may be used in print to mark the end of a proof. The two-cell symbol shown below is brailled, preceded and followed by a space. Note that the bar may be printed at the right margin.

Boldface Vertical Bar	❘	⠸⠳

Example 13.2-5   **Proof.** RA = ST because halves of equal quantities are equal.   ❘

⠠⠏�⠋⠲ ⠠⠗⠠⠁ ⠶ ⠠⠎⠠⠞ ⠃⠑⠉⠁⠥⠎⠑
⠿ ⠓⠁⠇⠧⠑⠎ ⠷ ⠑⠟⠥⠁⠇ ⠟⠥⠁⠝⠞⠊⠞⠊⠑⠎ ⠜⠑ ⠑⠟⠥⠁⠇⠲ ⠸⠳

PRACTICE 13B

Spaced Miscellaneous Symbols

1. Su bought 25 boxes of tissue for her classroom. Priced @99¢, can she pay with only one $20 bill?

$$25 \times \$.99 = \$24.75$$
$$\checkmark \$24.75 > \$20$$

   Answer: No. Su needs more than $20 to buy the tissues.

2. $\because 8x + 3y = 15$, substituting 0 for x gives $8(0) + 3y = 15$, or $3y = 15$. $\therefore y = 5$.

3. The end of proof mark " $\blacksquare$ " is used in mathematics to denote the end of a proof, in place of the traditional abbreviation "Q.E.D." for the Latin phrase "**quod erat demonstrandum**", meaning "which was to be shown".

4. Discuss the following table in your group.

BTB	ACR	pH range
5.55×10^{-5}	2.57×10^{-11}	1.00-6.39
4.75 "	2.33 "	1.49-12.01
5.30 "	3.24 "	0.99-12.31
4.44 "	2.06 "	1.17-12.01

Spacing with the Angstrom Unit and Tally Marks

13.3 Angstrom Unit: The angstrom unit is a mathematical symbol, but it is spaced according to the rules of abbreviations.

Angstrom Unit	Å	⠠⠠⠜

≫ 1Å ⠼⠁⠠⠠⠜

A space is left before and after the angstrom unit even when it is preceded or followed by a sign of operation.

Example 13.3-1 1Å + 2Å = 3Å

⠼⠁ ⠠⠠⠜ ⠖ ⠼⠃ ⠠⠠⠜ ⠐⠶ ⠼⠉ ⠠⠠⠜

A space is <u>not</u> left between the angstrom unit and a sign of grouping, a horizontal or diagonal fraction line, a braille indicator, or a punctuation mark which applies to it.

Example 13.3-2 An angstrom unit $\left(Å\right)$ equals 1×10^{-10} meter.

⠠⠁⠝ ⠁⠝⠛⠌⠗⠕⠍ ⠥⠝⠊⠞ ⠷⠠⠠⠜⠾ ⠑⠟⠥⠁⠇⠎ ⠼⠁⠈⠡⠼⠁⠴⠡⠤⠡⠴ ⠍⠑⠞⠑⠗⠲

The angstrom unit is unspaced from the opening and closing parentheses.

Example 13.3-3 $\left(\lambda > 7{,}000Å\right)$

⠦⠈⠇ ⠈⠕ ⠼⠛⠂⠴⠴⠴⠠⠠⠜⠴

A space precedes the angstrom unit, but it is unspaced from the closing parenthesis.

Example 13.3-4 $\dfrac{(2)\left(2Å\right)}{4\times10^6 Å}$

⠷⠼⠃⠾⠷⠼⠃⠠⠠⠜⠾ ⠌⠼⠙⠈⠡⠼⠁⠴⠡⠠⠠⠜ ⠌

A space precedes each angstrom unit, but the first angstrom unit is unspaced from the closing parenthesis and the second angstrom unit is unspaced from the closing fraction indicator.

Example 13.3-5 $(500 - 1000)Å$

⠷⠼⠑⠴⠴⠤⠼⠁⠴⠴⠴⠾ ⠠⠠⠜

The closing parentheses does not apply to the angstrom unit, therefore the symbol is preceded by a space.

13.3.1 Punctuation: Although the Angstrom unit is spaced like an abbreviation, it is a mathematical symbol and therefore must be punctuated mathematically when the punctuation falls within the Nemeth Code switches.

> ≫   1Å.   �

The angstrom unit is unspaced from the punctuation which follows it.
(Assume Nemeth Code continues after this period.)

13.4 Tally Mark

Tally Mark	|	⠇⠿

13.4.1 Grouping: If tally marks are grouped in print they are similarly grouped in braille, with one blank cell between groupings.

> ≫ ||||| ||||| |||

When a group of tally marks is crossed through, the cross-tally is treated as an additional (456) tally mark which is added to its group.

> ≫   ⅠⅠⅠⅠ ⅠⅠⅠⅠ |||

In the print copy, each group of five tally marks appears a long diagonal stroke superposed on four vertical strokes.

13.4.2 Spacing: One space is left before and after a group of tally marks even when the group is preceded or followed by a sign of operation.

> ≫ ||| + ||| = ||||||

No space is left between a group of tallies and a sign of grouping or a braille indicator applying to it.

> ≫   (||||||) + (|||)

> ≫   (||| + |||)

> ≫   ⅠⅠⅠⅠ ²

When the combination of symbols cannot be accommodated on one braille line, transition to another braille line may take the place of the required space.

13.4.3 Punctuation: When a tally mark is followed by punctuation requiring the punctuation indicator, a multipurpose indicator (dot 5) is placed between the tally mark and the punctuation indicator to avoid misreading the identical symbols. No space is left between a group of tallies and a punctuation mark applying to it.

➢ ⌊⊦⊦⊓, ⌊⊦⊦⊓. ⠿⠿⠿⠿⠿⠄ ⠿⠿⠿⠿⠿⠂⠶⠂

*See **Lesson 6** regarding spacing with the crosshatch symbol # and the diagonal slash /.*

PRACTICE 13C

I. Tally marks | | | | | | | equal the number 11_{six}. What numeral does | | | | | | | | | equal in base six?

II. Does | | | + | | | = ⌊⊦⊦⊓ | ?

III. $5550\text{Å} = 555m_\mu$ or 0.555 micron. (m_μ is a millimicron.)

SUPERPOSED SIGNS

13.5 Definition and Analysis: Superposed signs are signs which are printed one upon another so that one sign extends beyond the boundary of the other. Contrast this with "shapes with interior modification" presented in **Lesson 11**, where one symbol is printed inside the boundaries of the other. Here are some examples of superposed signs.

$$\oint \qquad \Subset \qquad \rightarrowtail \qquad \ll \qquad \not\angle$$
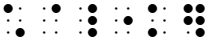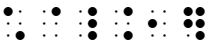
In order to transcribe a superposed sign, the basic sign and the superposed sign need to be determined because the basic sign is brailled first. The following order of preference is used as a guide. A symbol lower on the list is regarded as being superposed upon a symbol higher on the list.

— Integral sign
— Signs of operation
— Horizontal and vertical bars
— Signs of shape
— Signs of comparison
— Signs not listed above

Here is an analysis the first three print examples shown above.

$\oint$ The basic sign is an integral sign; the superposed sign is a sign of shape (circle).

$\Subset$ The basic sign is a sign of operation (dot); the superposed sign is a sign of comparison (inclusion).

$\rightarrowtail$ The basic sign is a vertical bar; the superposed sign is a sign of comparison (arrow).

If two signs belong to the same category, the superposition may be represented in either order, provided the same order is followed consistently throughout the transcription. Here is an analysis the last two print examples shown above.

    ≪   Both signs belong to the same category – signs of comparison (nested "less than" signs).

    ∡   Both signs belong to the same category – signs of shape (an angle and an arc).

13.6 Transcription of Superposed Signs: The components of a superposed sign are unspaced and are transcribed in the following order:

— The symbol for the basic sign is written first.
— The superposition indicator is written next.
— The symbol for the superposed sign is written third.
— The termination indicator is written last.

Superposition Indicator	⠘
Termination Indicator	⠰

13.6.1 Integral Modified by Superposition: The most common integrals with superposed symbols are listed below. Unlisted "integrals modified by superposition" are transcribed in accordance with the rules for superposed signs. Spacing and punctuation follow the same rules as for unmodified integral signs (see **13.1.8**).

Integral with superposed circle	$\oint$	⠮⠘⠿⠰
Integral with superposed infinity	$\oint$	⠮⠘⠿⠰
Integral with superposed rectangle	$\oint$	⠮⠘⠿⠰
Integral with superposed square	$\oint$	⠮⠘⠿⠰

≫ $\oint \dfrac{\mathrm{d}q}{T} < 0$

≫ $\oint P \,\mathrm{d}x + Q \,\mathrm{d}y$

≫ $\oint f(x, y) \,\mathrm{d}y$

13.6.2 Signs of Operation Modified by Superposition: The most common symbols use the multiplication dot as the basic symbol. Three examples are shown below.

Dot

    ... between bars of equals sign ⠀⠀⠀⠀ ⠩ ⠀⠀⠀⠀ ⠐⠣⠙⠣⠒⠒⠿

    ... within inclusion sign ⠀⠀⠀⠀ ⊂ ⠀⠀⠀⠀ ⠐⠣⠙⠣⠢⠣⠿

    ... within reverse inclusion sign ⠀⠀⠀⠀ ⊃ ⠀⠀⠀⠀ ⠐⠣⠙⠣⠒⠢⠿

⠀⠀⠀⠀⠀ $\pi \doteq 3.14$ ⠀⠀⠀⠀ ⠨⠏ ⠀ ⠐⠣⠙⠣⠒⠒⠿ ⠶⠒⠢⠲⠁⠙
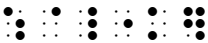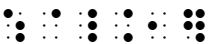
Unlisted "signs of operation modified by superposition" are transcribed in accordance with the rules for superposed signs.

13.6.3 Horizontal and Vertical Bars Modified by Superposition: The most common symbols are shown below.

Horizontal Bar

    ... through inclusion sign ⠀⠀⠀⠀ ⟞ ⠀⠀⠀⠀ ⠐⠣⠱⠣⠢⠣⠿

    ... through reverse inclusion sign ⠀⠀⠀⠀ ⟝ ⠀⠀⠀⠀ ⠐⠣⠱⠣⠒⠢⠿

Vertical Bar

    ... through shaft of right-pointing arrow ⠀⠀⠀⠀ → ⠀⠀⠀⠀ �

    .. through shaft of left-pointing arrow ⠀⠀⠀⠀ ← ⠀⠀⠀⠀

Unlisted "bars modified by superposition" are transcribed in accordance with the rules for superposed signs.

⠀⠀⠀⠀⠀ ⊖ ⠀⠀⠀⠀ ⠐⠣⠻⠫⠿

"Horizontal bar" is higher on the list than a sign of shape (the circle).

13.6.4 Signs of Shape Modified by Superposition

⪢   △   ⠰⠫⠲⠒⠻⠀⠫⠲⠒⠒
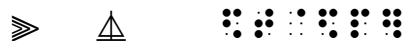
"Triangle" is a sign of shape; "perpendicular to" is a sign of comparison.

Signs of shape modified by superposition are spaced and punctuated as other signs of shape. (See **Lesson 11**).

When both signs are signs of shape, the superposition may be represented in either order, provided the same order is followed consistently throughout the transcription.

⪢   ∡   ⠫⠪⠫⠲⠒⠒ OR ⠫⠲⠒⠫⠪⠒⠒

*This arc shape extends beyond the boundary of the angle shape, making this "a shape modified by superposition". Compare this symbol to the "angle with interior arc" (**Lesson 11**) which has a different braille form.*

Note: When the print copy uses either "angle with interior arc" symbol throughout the text to simply mean "angle", the simple braille shape symbol for "angle" ⠫⠪ may be used. A transcriber's note is required to inform the reader of the substitution.

∠   △   ∡

13.6.5 Signs of Comparison Modified by Superposition: When both signs are signs of comparison, the superposition may be represented in either order, provided the same order is followed consistently throughout the transcription.

Two Signs of Comparison

    Equals Sign Through Inclusion Sign

                    $\in$ ⠐⠫⠩⠪⠱⠈⠸⠱

                    or   ⠸⠪⠱⠩⠪⠈⠱

    Equals Sign Through Reverse Inclusion Sign

                    $\supseteq$ ⠐⠫⠱⠪⠱�011⠘

                    or

    Nested Greater Than Signs (means "is large compared with")

      ... with straight sides $\gg$

      ... with curved sides   ⪢

    Nested Less Than Signs (means "is small compared with")

      ... with straight sides $\ll$

      ... with curved sides   ⪡

Spacing and punctuation follow the same rules as for any other sign of comparison (see **6.6**).

       13.6.6 Symbols That Are Not Superposed Signs: Negated symbols are not transcribed as superposed signs. See **Lesson 6** and **Lesson 11**. Also note that the print practice of showing a group of five tally marks as a long diagonal stroke superposed on four vertical strokes does not follow the Nemeth Code rules for superposed signs. Review section **13.4** above.

Instructions: See **Lesson 11** to review symbols with interior shapes and **Lessons 6**, **11**, and **13** regarding negated symbols.

PRACTICE 13D

Superposed Signs	Symbols with Interior Shapes	Negated Relations
———————	———————————	——————
$\oint$	⊙	$/\!\therefore$
Ψ	⊡	$\neq$
$\ominus$	$\ominus$	$\nparallel$
$\sphericalangle$C	⊿	$\nleqslant$
$5 \ll y$	$\angle_{45°}$	$\nleqslant$
R ⇀ s	⑬	$\neq$
Q $\nrightarrow$ R	⍟	$\slash$

AMBIGUOUS SIGNS

13.7 Context: It is often easy to mistake the print symbols for the letters o, O, and the numeral "zero", or letters l, i, I, and the numeral "one". Beyond these obvious examples, there are certain print signs which look similar to other print signs. The braille symbols may be altogether different. Brailling the wrong symbol will give the reader false information. In order to assure your transcription is correct, search the surrounding context to determine the meaning of the ambiguous sign. Magnification may help you identify the symbol. If you are still unsure, seek help from someone knowledgeable in the math or science topic who can correctly identify the print symbol. Some examples include ...

φ Greek letter phi ⠷⠨⠺	or	Φ null set ⠨⠹⠈⠚
	or	∅ canceled numeral zero ⠹⠈⠚⠼⠴
	or	0 zero in certain fonts ⠼⠴⠈⠚
	or	Φ horizontal bar with superposed circle ⠫⠉⠳⠻⠮
	or	θ Greek letter theta ⠨⠹
α Greek letter alpha ⠨⠁	or	∝ "varies as" ⠰⠰⠴
	or	*a* English letter "a" ⠸⠁
ε Greek letter epsilon ⠨⠑	or	Ɛ "membership" ⠈⠑
ν Greek letter nu ⠨⠝	or	*v* English letter "vee" ⠨⠧
Δ Greek letter Delta ⠨⠠⠙	or	△ triangle shape ⠫⠙
	or	Δ logical product with underbar ⠈⠩⠱
< "less than" ⠐⠅	or	⟨ opening angle bracket ⠨⠨⠣
	or	< left-pointing caret ⠈⠣
> "greater than" ⠐⠂	or	⟩ closing angle bracket ⠨⠨⠜
	or	> right-pointing caret ⠈⠜
|| two vertical bars ⠳⠳	or	|| "is parallel to" ⠫⠇
^ mathematical caret ⠘⠲	or	∧ logical product ⠈⠩
	or	^ literary (UEB) caret ⠘⠢ or circumflex ⠘⠩
√ radical sign ⠜	or	✓ checkmark ⠈⠜

¬   Is it a right-pointing arrow with lower-only, straight arrowhead ⠳⠒⠒⠕⠖

           ... or a long-division structure

           ... or the negation symbol (brailled using a transcriber-devised symbol)

13.7.1 Vertical Bar and Colon: The symbols that give transcribers the most trouble due to their ambiguity are the vertical bar and the colon. The vertical bar can be a grouping sign, an operation sign, a comparison sign, or an "end of proof" symbol. The colon can be a ratio symbol or a punctuation mark. You need to recognize the meaning of the sign in order to braille the proper symbol.

	(vertical bar)		
	... as a grouping sign	⠇	
	... as a sign of operation	⠇	(means "is a factor of")
	... as a sign of comparison	⠀⠇⠀	(means "such that" or "given")
	... as an "end of proof" symbol	⠸⠇	(printed in boldface)

:	(colon)		
	... as a mathematical symbol	⠐⠂	(ratio, meaning "is to"; "such that", mapping notation, and other meanings)
	... as a punctuation mark	⠸⠒	(preceded by a punctuation indicator where appropriate)

13.7.2 Spacing: Some signs use the same braille symbol but have different spacing rules depending on their meaning. You can't depend upon the print copy to show the spacing according to Nemeth Code rules so you need to recognize the meaning of the sign in order to apply proper spacing.

|	⠇	Is the vertical bar a grouping sign, an operation sign, or a comparison sign?	
~	⠈⠔	Is the tilde an operation sign ("not") or a comparison sign ("is related to" or "is similar to")?	
'	⠄	Is it a punctuation sign (an apostrophe or single quotation mark) or is it a math symbol (prime sign)?	
/	⠸⠌	Is it mathematical (a fraction line) or a UEB solidus?	
:	⠐⠂	A spaced sign of comparison, or	
	⠸⠒	An unspaced symbol?	

13.7.3 Uppercase Greek Letters: Some uppercase Greek letters are indistinguishable from English letters. Unless the text identifies the letter as Greek, you can safely assume it is an English letter.

13.7.4 Chemical Notation: Some signs have yet another meaning in chemical notation. For example, the following symbols can be certain types of chemical bonds:

$$= \ || \ \equiv \ — \ | \ / \ \backslash \ \cdot \ : \ ::$$

Details can be found in the *Braille Code for Chemical Notation*.

MULTIPURPOSE INDICATOR

> **Multipurpose Indicator** ⠐

13.8 Review: In addition to being a baseline indicator, braille dot 5 assumes several other functions in the Nemeth Code. Dot 5 is called the *multipurpose indicator* in the following situations already introduced in previous lessons:

— A multipurpose indicator is used between two unspaced signs to indicate that they are <u>printed horizontally</u> (side-by-side).

    – side-by-side signs of operation. See **6.2**.

    – side-by-side tildes. See **6.4.9**.

    – side-by-side signs of comparison. See **6.9**.

    – side-by-side arrows. See **10.14**.

    – side-by-side modifiers within a sign of shape. See **11.17**.

— A multipurpose indicator is used when a <u>number follows a letter</u> with no space between them. See **7.16**.

— A multipurpose indicator is used between a <u>regular polygon</u> representing a sign of operation <u>and a numeral</u> immediately following it. See **11.29**.

— A multipurpose indicator triggers the beginning of a <u>modified expression</u>. See **12.2**.

— A multipurpose indicator is placed between a <u>tally mark</u> and a following <u>punctuation indicator</u> to avoid misreading the similar symbols. See **13.4.3**.

13.9 Additional Uses of the Multipurpose Indicator

13.9.1 Letter Followed by a Decimal Point and a Numeral: When a letter on the baseline of writing is immediately followed by a decimal point and a numeral, a multipurpose indicator is placed between the letter and the decimal point to show that the decimal point and numeral are not subscripts to the letter.

    ⟫ x.4   ⠭⠐⠲⠼⠙

13.9.2 Numeric Subscript Followed by a Numeral: A multipurpose indicator is used after a numeric subscript if the subscript is followed by a numeral on the baseline of writing.

    ⟫ $x_7 10$ ⠭⠐⠼⠛⠐⠼⠁⠚

13.9.3 Decimal Point Followed by a Nonnumeric Character: A multipurpose indicator is used after a decimal point if the symbol following the decimal point is something other than a numeral.

    ⟫ $0. a_1 a_2 a_3 \dots a_n$

    ⟫ $\dfrac{1}{2} = .\ \underline{?}$

    ⟫ (5.)

Note: When a decimal point is followed by a comma or the punctuation indicator, a multipurpose indicator is not needed.

Example 13.9-1   Can you explain the significance of the decimal point in 1., 10., and 100.? 2., 20., and 200.?

⠠⠉ ⠐⠁⠝ ⠽⠳ ⠑⠭⠏⠇⠁⠔ ⠮ ⠎⠊⠛⠝⠊⠋⠊⠉⠁⠝⠉⠑ ⠷ ⠮ ⠙⠑⠉⠊⠍⠁⠇

⠏⠕⠔⠞ ⠔ ⠼⠁⠲ ⠼⠁⠚⠲ ⠯ ⠼⠁⠚⠚⠲⠦ ⠼⠃⠲

⠼⠃⠚⠲ ⠯ ⠼⠃⠚⠚⠲⠦

13.9.4 Side-by-Side Vertical Bars: A multipurpose indicator is used between two unspaced vertical bars when the first bar is a closing sign of grouping, and the second bar is an opening sign of grouping.

> $|x||y|$ ⠳⠭⠳⠐⠳⠽⠳

A multipurpose indicator is used between two vertical bars which are nested grouping symbols (one bar is shorter and/or thicker than the other).

> $||x||$ ⠳⠐⠳⠭⠳⠐⠳

This example shows only one outer closing bar.

> $|x||_{x=0}$ ⠳⠭⠳⠐⠳⠱⠭ ⠀⠭⠀⠚

Instructions: Explain the function of the multipurpose indicator in each example.

PRACTICE 13E

1. x3, R10   ⠿⠒ ⠶⠶ ⠿   ⠶⠆⠿ ⠶⠶ ⠿

2. $120° + n320°$

3. $C = \pi 2r$

4. $\omega 2 = \omega \neq 2\omega$

5. $140te4t5_{12} + e5_{12}$

6. $\frac{A_0}{2} = A_0 2^{-0.05T}$

7. $x_2 = n_1 5^{-1} - 1n_2 5^{-1}$

8. $0.\alpha_1\alpha_2\alpha_3 \dots \alpha_n$

9. .%

10. $4\% = .\underline{\qquad}$

11. $5. + .6 = 5.6$

12. $\|x\|\|y\|$

13. $\|\|x\|\|$

DIVISION PROBLEMS

13.10 Linear Representation: Lesson 1 introduced linear division problems using the symbol ÷ for "divided by", as in 15 ÷ 3 ("fifteen divided by three"). A division problem may also be printed using a curved, straight, or slanted line for the division sign, and a horizontal line printed above or below the divisor.

$$3\overline{)15}$$

When only a divisor and dividend are printed, and when the divisor and dividend are composed entirely of numerals with or without decimal points and/or commas, the problem is brailled as a linear construction. The print style is replicated by using one of the following symbols between the divisor and dividend. Note that the straight and slanted division signs are represented with the same braille symbol.

Division Signs:		
Curved Right	⠌	)
Curved Left	⠳	(
Straight or Slanted	⠩	| or \ or /

In this linear format, only the division sign is brailled. The horizontal line printed above or below the dividend is not brailled. The numeric indicator is used where required according to other rules of the Nemeth Code. As with other operation signs, the division symbol is unspaced from the divisor and the dividend even if the print copy shows a space.

≫   3) 15   �

≫   3 ⌊ 15

≫   110 (10

A horizontal line printed below the divisor also is disregarded.

≫   30 ⌉ 90

Example 13.10-1   Show how to solve $9\overline{)216}$ on your calculator. Is there a remainder?

Example 13.10-2   1. $3\overline{)15}$ 2. $0.3\overline{)30}$ 3. $3\overline{)4.5}$ 4. $30\overline{)9,060}$

�...

(braille cells representing the division problems)

Reminders: When the print copy arranges nonspatial itemized material side-by-side across the page and there are no subdivisions, the braille format is changed so that all identifiers start in cell 1.To assure that identifiers each begin in the same cell, the opening Nemeth Code indicator is on the line before the first numbered item.

Instructions: Use appropriate margins for embedded, displayed, and itemized problems.

PRACTICE 13F

Converting Fractions to Decimals

To write the fraction 1/2 as a decimal you can divide "one point zero divided by two" or $2\overline{)1.0}$. Put away your calculator! Using mental math, it's easy! The answer is .5. Can you divide *these* fractions in your head?

1/5: $5\overline{)1.0}$

4/5: $5\overline{)4.0}$

9/10: $10\overline{)9.0}$

Practice

1. Solve these division problems.

a. $8\overline{)128}$ b. $6\overline{)5,304}$

c. $4\overline{)204}$ d. $9\overline{)837}$

SPATIAL ARRANGEMENT WITH DIVISION

13.11 Notation Devices: It will be helpful to refer to the parts by using the following standard terminology. In this example ("forty divided by ten equals 4"), "40" is the *dividend,* "10" is the *divisor,* and "4" is the *quotient*.

$$\text{divisor}\overline{\smash{)}\text{dividend}}^{\text{quotient}} \qquad 10\overline{\smash{)}40}^{\,4}$$

Other print styles which can be found in various publications are illustrated below using the same numbers for dividend, divisor, and quotient. Each style includes a division sign (either straight, slanted, or curved) and a separation line. The quotient can be printed above, below, or next to the dividend.
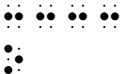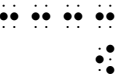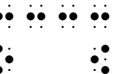
In a spatial transcription the separation line (the horizontal line between the dividend and the quotient) is brailled. The length of the line varies depending on the layout of the problem.
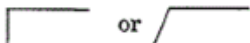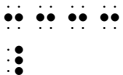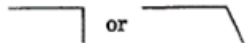
The spatial division constructions given in the Nemeth Code are shown below. Notice the similarity between the print sign and the braille symbols.

Separation Line Above the Dividend

Curved Division Sign

on Left	on Right	on Left and Right

Straight or Slanted Division Sign

on Left	on Right

Straight Division Signs on Right and Left

Separation Line Below the Dividend

Curved Division Sign
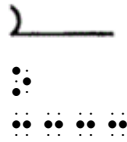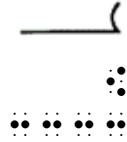
on Left	on Right	.on Left and Right
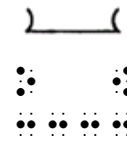
Straight or Slanted Division Sign
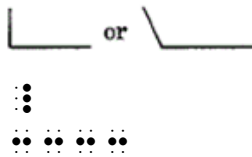
on Left	on Right
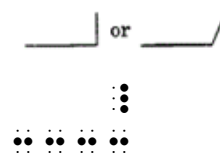
Straight Division Signs on Left and Right

13.11.1 Length of the Separation Line: As shown above, the separation line which appears in a spatially-arranged division problem begins in the column containing a division symbol and ends in the column containing the other division symbol if the latter appears in print. When only one division symbol is printed, the separation line ends in a cell one column beyond the overall arrangement. Notice how this rule applies to the examples in the rest of this section.

Review of Format for Spatial Arrangements

- A blank line is required above and below a spatial arrangement.

- The numeric indicator is not used. (There are some exceptions.)

- Side-by-side arrangement is allowed.

- An identifier associated with a spatial problem is positioned according to rules applying to the type of arrangement, not necessarily at the top line of the problem. (See **13.17** regarding spatial division.)

13.12 When a Spatial Arrangement is Required: When more than just the divisor and dividend are shown, or when the divisor and/or dividend consist of more than just numerals, a spatial (vertical) arrangement is required.

In this section, code switch indicators are not shown in the examples which contain no narrative.

13.12.1 More Than Just Numerals: The presence of the dollar sign in the dividend in the next example means a spatial arrangement must be brailled. *The required blank line before and after the arrangement is not shown in the isolated examples.*

Example 13.12-1   40 | $49.99

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

Reminders: The division symbol is unspaced from the divisor and the dividend. The separation line begins in the cell directly over the division sign. The separation line ends one cell beyond the overall arrangement.

Because the next example contains abbreviations in the dividend, a spatial arrangement is required.

Example 13.12-2   6) 1 ft. 5 in.

Reminder: The numeric indicator is not used in a spatially-arranged division problem.

13.12.1.a Spacing with Abbreviations: Spacing rules for abbreviations next to operation signs must be applied, as illustrated in this example where the dividend is printed to the left of the division symbol.

Example 13.12-3   1 ft. 5 in. (6

Reminder: A space is required between an abbreviation and a sign of operation.

13.12.2 A Quotient is Present: The quotient is aligned with the dividend (as shown in the print copy) unless it has been intentionally misaligned as an exercise for the student.

Example 13.12-4   5,000
                    10) 50,000

The next example shows a different print arrangement of the division sign and separation line.

Example 13.12-5   110 (10
                        11

Reminders: One end of the separation line is directly aligned with the division sign; the separation line extends one cell beyond the overall arrangement.

In the next print style, the division sign is a straight line, the separation line is above the dividend, and a horizontal line is printed under the divisor. The line under the divisor is not shown in the braille transcription.

Example 13.12-6

$$\begin{array}{r} 5{,}000 \\ \hline 10\overline{)50{,}000} \end{array}$$

⠰⠂⠀⠒⠀⠢⠀⠢⠀⠢
⠿⠿⠿⠿⠿⠿⠿⠿
⠈⠢⠀⠢⠩⠑⠀⠢⠀⠒⠀⠢⠀⠢

The same example is shown in different print style. Two division symbols are printed, and the quotient is printed to the right of the dividend rather than above the separation line. The separation line begins at the left division sign and ends at the right division sign.

Example 13.12-7 $10 \,\overline{)\, 50{,}000 \,(}\, 5{,}000$

⠿⠿⠿⠿⠿⠿⠿⠿⠿
⠈⠢⠒⠾⠑⠢⠾⠒⠾⠾⠾⠷⠾⠈⠢⠢⠒⠾⠾

> *Reminder: Because a quotient is shown, this problem is brailled as a spatial arrangement. The divisor, dividend, and quotient are brailled unspaced from the division symbols.*

13.12.2.a A Quotient with a Remainder: When a remainder is printed next to the quotient, the capitalized or uncapitalized "r" (indicating "remainder") is preceded by a space. A multipurpose indicator (dot 5) separates the letter from the numeral following it.

Example 13.12-8 $37 \,\overline{)\, 476 \text{ sq. in.} \,(}\, 12 \text{ R32}$

⠿⠿⠿⠿⠿⠿⠿⠿⠿⠿⠿⠿⠿⠿⠿⠿
⠿⠒⠫⠛⠿⠶⠀⠼⠙⠛⠋⠀⠎⠟⠲⠀⠔⠝⠲⠀⠾⠼⠁⠃⠀⠠⠗⠐⠼⠉⠃

When the quotient is arranged above the separation line, the line extends one cell to the right of the overall arrangement, including the remainder.

Example 13.12-9

$$\begin{array}{r} 18 \text{ r2} \\ \hline 25 \mid 452 \end{array}$$

⠀⠼⠁⠓⠀⠀⠗⠼⠃
⠿⠿⠿⠿⠿⠿⠿⠿
⠼⠃⠑⠐⠌⠼⠙⠑⠃

13.12.3 Long Division: In a long division problem, multiples are written below the dividend. A series of subtraction problems are performed, giving partial remainders and, finally, a remainder. The long division portion of the problem (bracketed below) is aligned for subtraction.

```
        123
  17 ) 2091      dividend
        17       multiple (17×1)  ⎤
        39       partial remainder │
        34       multiple (17×2)   ⎬  subtraction problems
        51       partial remainder │
        51       multiple (17×3)   │
         0       remainder        ⎦
```

In braille, the components of the problem are aligned in the same manner as they are aligned in the print copy. All separation lines must be the same length, regardless of their relative lengths in the print copy.

Example 13.12-10

```
        123
  17 ) 2091
        17
        39     ⎤
        34     │
        51     ⎬  aligned for subtraction
        51     │
         0     ⎦
```

Example 13.12-11

```
        18 r2
  25 ) 452
        25
       202
       200
         2
```

Example 13.12-12

$$x-3 \overline{\smash{)}\ x^2 + x - 12} \quad \begin{array}{r} x + 4 \end{array}$$

$$\begin{array}{r} x^2 - 3x \\ \hline 4x - 12 \\ 4x - 12 \\ \hline 0 \end{array}$$

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
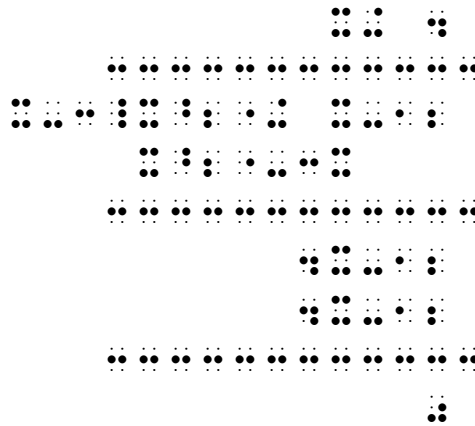
When commas or decimal points occur in a dividend, corresponding blank cells should be left throughout the body of the division example, except in the separation lines.

Example 13.12-13

$$36 \overline{\smash{)}\ \$37.80} \quad \begin{array}{r} \$\ \ 1.05 \end{array}$$

$$\begin{array}{r} 36 \\ \hline 1\ 80 \\ 1\ 80 \end{array}$$

Example 13.12-14

$$27 \overline{\smash{)}\ 1{,}352.42} \quad \begin{array}{r} 50.09 \end{array}$$

$$\begin{array}{r} 1\ 35 \\ \hline 2\ 42 \\ 2\ 42 \end{array}$$

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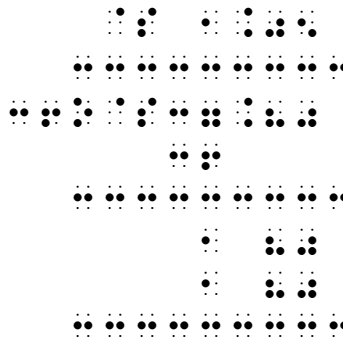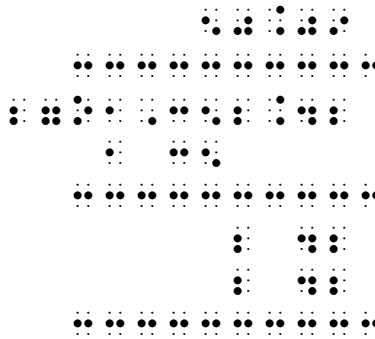
If a caret occurs in a dividend, corresponding blank cells are left throughout the body of the division problem (except in the separation lines). Since two cells are required for the caret, the decimal point replacing the caret in the quotient is placed in the right-hand cell.

Example 13.12-15

$$2.5_{\wedge} \overline{)\, 28.7_{\wedge}0} $$

<pre>
        1 1.4
2.5ʌ | 28.7ʌ0
        25
        37
        25
       120
       100
        20
</pre>

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

If the quotient has been intentionally misaligned as an exercise for the student, the same misalignment is shown in the transcription.

Example 13.12-16   Find the mistake.

$$15 \overline{)\, 1635} $$

<pre>
      109
15) 1635
     15
     135
     135
</pre>

*The leftmost character in the spatial arrangement is placed in the appropriate cell for displayed material—in this case, cell 3. (See **Lesson 8** for details regarding margins for displayed mathematical expressions.) Review **10.38** regarding the placement of code switch indicators with displayed spatial arrangements.*

13.13 Blank Lines Required: A blank line must precede and follow a spatially-arranged problem, even when the problem is embedded within narrative.

Example 13.13-1   Jeremy writes the quotient next to the dividend: $10\overline{)50{,}000}\ (5{,}000$.

When would this writing style be useful?

⠀⠄⠸⠹⠑⠍⠽⠀⠶⠗⠊⠞⠑⠎⠀⠮⠀⠟⠥⠕⠞⠊⠑⠝⠞⠀⠝⠑⠭⠞⠀⠖⠕⠀⠸

⠀⠀⠒⠒⠒⠒⠒⠒⠒⠒⠒

⠮⠲⠙⠊⠧⠊⠙⠑⠝⠙⠒�⠶⠼⠑⠚⠂⠚⠚⠚⠀⠶⠐⠣⠀⠼⠑⠂⠚⠚⠚

⠀⠤�⠀⠎⠞⠽⠇⠑⠀⠃⠑⠀⠥⠎⠑⠋⠥⠇⠢
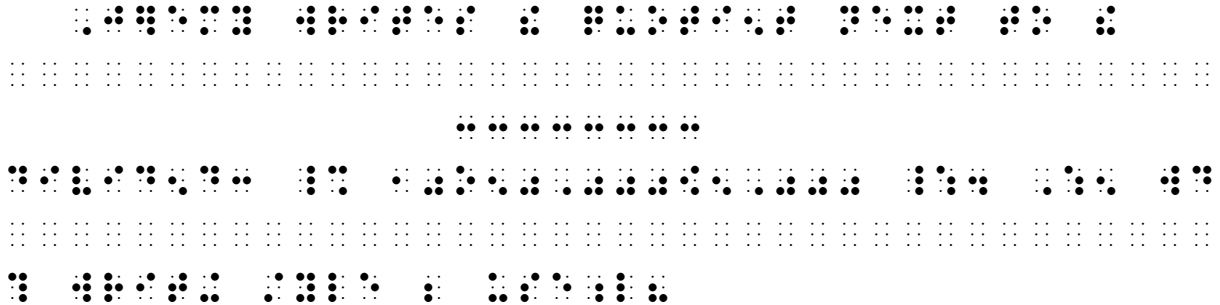
Instructions: Braille the first set of displayed problems as follows. Apply *Braille Formats* guidelines for a simple list in columns. The leftmost cell of column one is placed in cell 1; two blank cells come between the end of the longest item in one column and the left-hand margin of the next column. Guide dots are not used between unrelated columns. Maintain the same arrangement in braille (three columns) even though the fourth problem could fit next to the third problem. The last problem is displayed mathematical material.

<div align="center">

PRACTICE 13G

</div>

Here are six other ways to write $11\overline{)2233}$ (quotient 203) or "2233 divided by 11 equals 203":

$$\frac{203}{2233}\ (11 \qquad\qquad 11/\overline{2233}^{\,203} \qquad\qquad 11\overline{|2233}^{\,203}$$

$$\overline{2233|}\ 11 \qquad\qquad \overline{2233(}\ 11 \qquad\qquad 11\ )\ \overline{2233}$$
$$\quad\ 203 \qquad\qquad\quad\ 203 \qquad\qquad\qquad 203$$

What is $11\overline{)2234}$? Use long division.

$$
\begin{array}{r}
203\ R1 \\
11\overline{)2234} \\
\underline{22} \\
34 \\
\underline{33} \\
1
\end{array}
$$

13.14 Omissions: When the arrangement contains omissions, in order to maintain necessary alignment *only the general omission symbol is used*. Other omission symbols such as the long dash, the ellipsis, or a sign of shape are not used, even when the omissions are shown using these symbols in the print copy.

General Omission Symbol	⠿

Example 13.14-1

$$7\overline{)\,49}\quad\square$$

The same number of omission symbols as shown in print should be used.

Example 13.14-2

$$6\overline{)72}\quad ??$$

A shaded or unshaded region that does not show a specific number of places is represented by a single general omission symbol, aligned above the rightmost character in the dividend.

Example 13.14-3

$$6\overline{)72}\quad \blacksquare$$

13.15 Carried Numbers in Long Division: Print copy does not have a standard layout for carried numbers in a division arrangement. When carried numbers are shown, regardless of print placement it is suggested that the transcriber arrange the carried numbers below the carried number indicator and to the left of the dividend digits to which they apply. A transcriber's note describing both the print and braille formats must be included.

Carried Number Indicator (varying in length)	⠒⠒⠒⠒⠒⠒

The carried number indicator is to be the same length as the separation line. Blank spaces are left in the dividend to allow separate columns for the carried numbers. No blank spaces are left in the carried number line.

Example 13.15-1

$$\begin{array}{r} 7\ 6\ 9 \\ 7\overline{)5\,3_4\,8_6\,3} \end{array}$$

> *The transcriber's note relating to this example could say "In print the carried numbers appear below and to the left of the dividend digits to which they apply. In braille, the same alignment is followed. A carried number indicator is brailled below the dividend."*

13.16 Cancellation in Long Division: If cancellation is shown in long division, the canceled material is enclosed in braille cancellation indicators. Blank cells are left where necessary for proper alignment.

Opening Cancellation Indicator	⠠⠷
Closing Cancellation Indicator	�

Example 13.16-1

$$\begin{array}{r} 5 \\ 78\!\!\!\diagup \\ \hline 65\,)\overline{5100} \\ 455 \\ \hline 550 \\ 520 \\ \hline 300 \\ 260 \\ \hline \cancel{40} \end{array}$$

The material canceled in braille must exactly represent what is canceled in print.

Example 13.16-2

$$\begin{array}{r} x - 5 \\ x + 2\,)\,\overline{x^2 - 3x - 10} \\ \underline{-x^2 - 2x} \\ \cancel{-5x - 10} \\ \cancel{+5x + 10} \\ \hline 0 \end{array}$$

Notes

"x–5" is placed as in print, in the first
    cell available to the quotient.

The division sign is unspaced from the divisor.

The minus sign is brailled in the same column
    (cell) as the division sign.
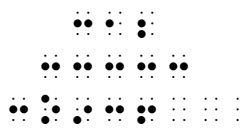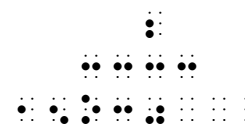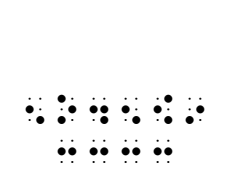
Space inserted above cancellation indicators.

Aligned for subtraction.

"0" is aligned by place value.

13.17 Placement of Identifiers with Spatial Division: An identifier, if present, is placed on the line with the dividend. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement including separation lines. When arranged side-by-side across the width of the page, no symbol of one spatial arrangement or its identifier may be less than three cells distant from any symbol on any line of a neighboring arrangement or its identifier, except at the ends of separation lines.

Example 13.17-1   1. $3\overline{)936}$ with quotient 312 2. $15\overline{)30}$ with quotient 2 3. $5\underline{)45}(9$

Instructions: Include the required transcriber's note regarding the carried numbers.

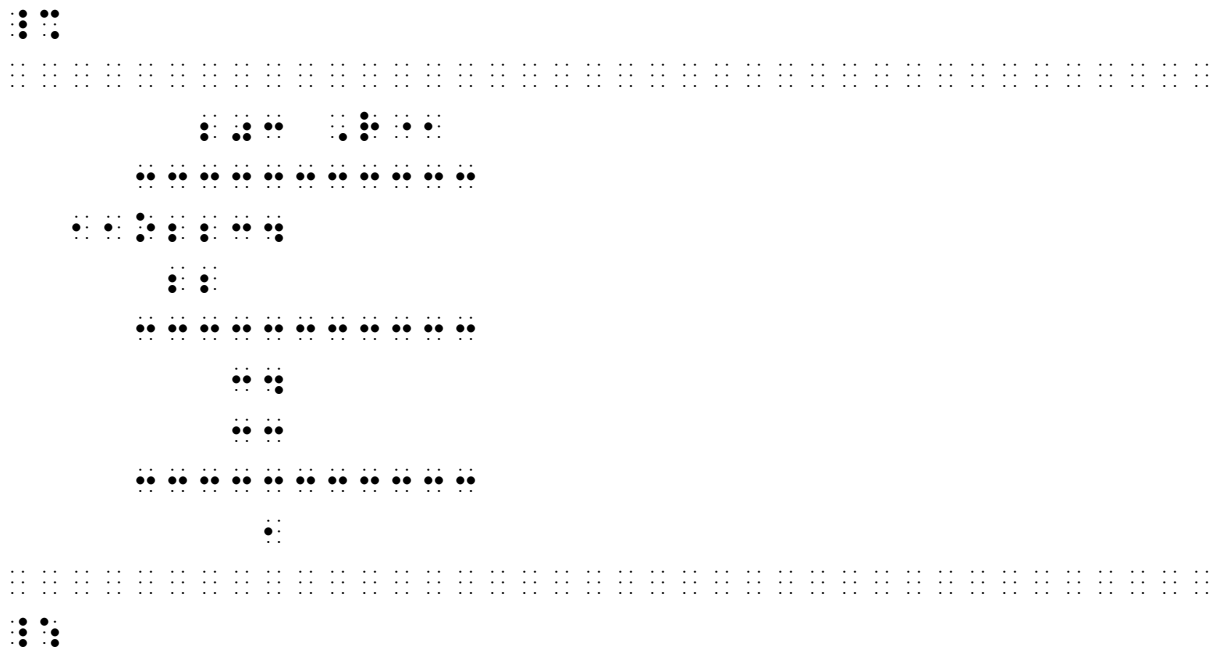## PRACTICE 13H

Fill in the boxes with the correct numeral.

(a) $6\overline{)23^55^1813}$ with quotient $3\ 9\ 302\ R\,\blacksquare$

(b) $25\overline{)80}$ with quotient $3\frac{1}{5}$, $3\frac{5}{25}$, 75, $\blacksquare$

(c) $4\overline{)19}$ with quotient $\blacksquare\ R3$

Other print layouts showing division will be taught in the next lesson.

For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL

Switch decision: Preferred practice is to braille grouping symbols in the same code, if possible. In item III, the opening and closing parentheses are in UEB. The opening Nemeth Code indicator is placed immediately after the opening parentheses.

Guide dots are not used because the column entries are not related across the line.

PRACTICE 13E

1. A multipurpose indicator is used when a letter is followed by a numeral and they are both on the baseline of writing.

2. The first dot 5 is a baseline indicator because the plus sign is on the baseline and it follows a raised hollow dot. The second dot 5 is a multipurpose indicator which is needed to show that the numeral "3" is not a subscript to the letter "n".

3. The same rule applies to letters in any alphabet – a multipurpose indicator is needed to show that the numeral "2" is not a subscript to the Greek letter pi.

4. Same as #3 regarding Greek letter omega followed by numeral "2" in "ω2". Note that a multipurpose indicator is not needed for a letter following a numeral, as in " 2ω".

5. A baseline indicator precedes the plus sign, following the subscript "12". (The multipurpose indicator is not used following the "t" and "e" because they represent numerals in base 12.)

6. A multipurpose indicator is needed after the second numeric subscript "0" because the subscript is followed by a numeral on the baseline of writing ("2").

7. The function of the first and third dot 5 is the same as item 6. The second dot 5 is a baseline indicator which is needed for the minus sign following a superscript.

8. A multipurpose indicator is needed after the decimal point because the next symbol is not a numeral—it is the Greek letter "alpha".

9. A multipurpose indicator is needed after the decimal point because the next symbol is not a numeral—it is a percent sign.

10. A multipurpose indicator is needed after the decimal point because the next symbol is not a numeral—it is a long dash.

11. A multipurpose indicator is needed after the decimal point because the next symbol is not a numeral—it is a plus sign.

12. A multipurpose indicator is used between two unspaced vertical bar symbols (in this case, each is a "double vertical bar" symbol) when the first is a closing sign of grouping, and the second bar is an opening sign of grouping.

13. A multipurpose indicator is used between vertical bars which are nested grouping symbols. (The double bar symbols are shorter than the single bar symbols.)

Reminder: The subdivisions each begin a new line in braille. The rule for non-spatial material states that <u>all</u> subdivisions must fit on one line in order to braille them side-by-side.

Lines 4 and 8 are blank because a spatial arrangement must be preceded and followed by a blank line, even when embedded within a narrative paragraph.

The Nemeth Code terminator is placed in cell 1 following the blank line after the completion of the spatially-arranged material.

This practice continues on the next page.

� braille content...

The opening Nemeth Code indicator appears on the same page as the material to which it applies. The leftmost character in the problem (line 5, above) is placed in cell 3, the margin for material displayed to 3-1 narrative.

Notes to Practice 13H

The instructions begin in cell 5 and are followed by the required transcriber's note which reads as follows: In print, carried numbers appear above and to the left of the dividend digits to which they apply. In braille the carried numbers appear *below* and to the left of the dividend digits to which they apply, beneath the carried number indicator.

A blank line precedes each spatial problem. Because the general omission symbol is non-numeric (even though it may represent a numeral), a multipurpose indicator is not needed following the letter "R" in problem (a).

EXERCISE 13

Exercise 13 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 14.

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