

LESSON 10

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

ARROWS

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INTRODUCTION TO SPATIAL ARRANGEMENTS

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ARROWS

10.1 Arrows Used in Mathematics: Here are some examples of arrows which bear mathematical meaning.

$$\begin{array}{cccc} A \rightarrow B & \overrightarrow{PQ} & 2CO \rightleftharpoons CO_2 + C & N \rightsquigarrow M \\ f: a \mapsto b & P \uparrow Q & A \rightarrow B \leftrightarrow C \leftarrow D & \end{array}$$

Construction of Braille Arrows

10.2 Introduction to the Shape Indicator: The shape indicator is brailled before an arrow symbol.

Shape Indicator ⠠

10.3 Horizontal Arrow Shafts: The length of a shaft is indicated by the number of times the braille arrow shaft symbol is used. Two braille symbols represent the ordinary shaft length for the particular publication. By comparison, one braille symbol represents a short shaft, and three or more symbols indicate a longer shaft.

Arrow Shafts

Ordinary length, single	—	⠠⠠
Short, single	—	⠠
Long, single	————	⠠⠠⠠

Arrow Shafts, cont.		
Ordinary length, double	==	⠠⠠
Short, double	=	⠠
Long, double	===	⠠⠠⠠
Dashed	--	⠠⠠
Dotted	...	⠠⠠⠠
Wavy	~~~~	⠠⠠⠠

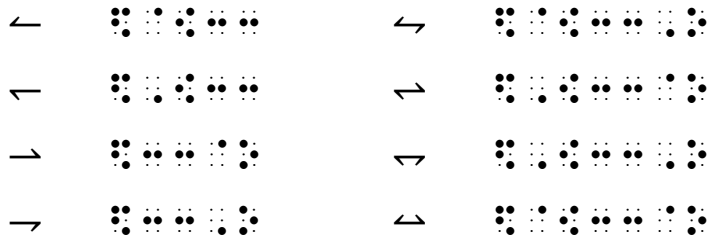
10.4 Arrowheads: An arrowhead may be barbed, blunted, straight, or curved, and may occur at the left, at the right, or at both ends of an arrow shaft. An arrowhead may also appear with only its upper or lower barb.

10.4.1 Barbed Arrowheads: This list shows the barbed arrowheads. Rule XXI of the Nemeth Code lists more arrowhead styles and the symbols used to represent them: blunted, curved, and straight arrowheads. (The shortest line in the illustrations below represent the end of a shaft.)

<u>Barbed Arrowheads</u>		
Full, left-pointing	←	⠠
Full, right-pointing	→	⠠
Lower only, left-pointing	↙	⠠⠠
Lower only, right-pointing	↘	⠠⠠
Upper only, left-pointing	↖	⠠⠠
Upper only, right-pointing	↗	⠠⠠

10.5 Arrows With Barbed Ends: Consider the shaft in the barbed arrows shown below to be the standard length for the book. A two-cell "ordinary length" shaft is brailled. Notice the similarity between the print and braille symbols. (*Note:* Code switch indicators are omitted in the isolated examples in this section.)

→	⠠⠠	⇒	⠠⠠⠠
←	⠠⠠	⇐	⠠⠠⠠
↔	⠠⠠⠠	↔	⠠⠠⠠⠠

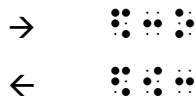


Note: An additional rule may apply to the first arrow shown above (\rightarrow). See [10.7](#), below.

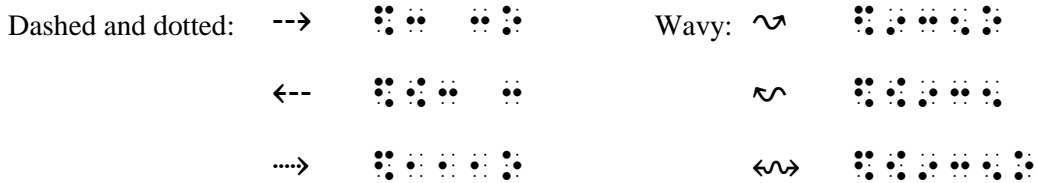
By comparison, the shafts in these barbed arrows are longer. A three-cell shaft is brailled.



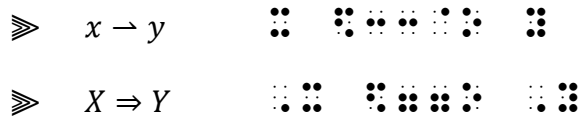
And, by comparison, the shafts in these barbed arrows are shorter. A one-cell shaft is brailled.



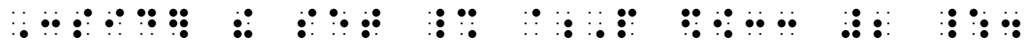
Other shaft styles are illustrated below.



10.6 Spacing and Punctuation with Arrows: When arrows function as a sign of comparison they are spaced according to the rules governing comparison signs.



Example 10.6-1 Consider the set $\mathcal{P} \leftarrow 2$.



The script letter P requires an English letter indicator even though the letter precedes a sign of comparison. (A typeform indicator must always be followed by an alphabetic indicator.)

When not functioning as a sign of comparison, an arrow is spaced according to its context. Punctuation associated with an arrow is mathematical.



Instructions: Leave one space (one blank cell) between the arrows in A. and B. Assume shafts in items A, C, and D to be of regular length, and in item B to be long. Assume all to be in regular type even though they may appear darker. In item D, divide the long expression before the arrow that comes between the double parentheses.

PRACTICE 10A

Arrows with Barbed Ends

- A. $\Leftrightarrow \rightarrow \leftrightarrow \leftarrow \dots \rightarrow \dashrightarrow \curvearrowright \hookrightarrow$
- B. $\leftarrow \longleftarrow \longleftrightarrow \rightleftarrows$
- C. The principal operator of the left formula in Fig. 7.2 is \leftrightarrow , while the principal operator of the right formulas is \rightarrow .
- D. Construct a truth table for $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$.

10.8 Arrows With Dotted Ends: Arrows may also be represented by a shaft preceded or followed by a solid dot or by a hollow dot.





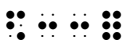
















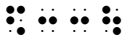

















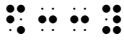














Solid Dot	•	⠠⠠⠠⠠⠠⠠
Hollow Dot	◦	⠠⠠⠠⠠⠠⠠⠠⠠

An assortment of possibilities is shown below.

◦	⠠⠠⠠⠠⠠⠠	◦	⠠⠠⠠⠠⠠⠠⠠⠠
◦	⠠⠠⠠⠠⠠⠠	◦	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
◦	⠠⠠⠠⠠⠠⠠⠠⠠	◦	⠠⠠⠠⠠⠠⠠
◦	⠠⠠⠠⠠⠠⠠	◦	⠠⠠⠠⠠⠠⠠
◦	⠠⠠⠠⠠⠠⠠	◦	⠠⠠⠠⠠⠠⠠⠠⠠
◦	⠠⠠⠠⠠⠠⠠	◦	⠠⠠⠠⠠⠠⠠⠠⠠
◦	⠠⠠⠠⠠⠠⠠	◦	⠠⠠⠠⠠⠠⠠
◦	⠠⠠⠠⠠⠠⠠	◦	⠠⠠⠠⠠⠠⠠
◦	⠠⠠⠠⠠⠠⠠	◦	⠠⠠⠠⠠⠠⠠
◦	⠠⠠⠠⠠⠠⠠	◦	⠠⠠⠠⠠⠠⠠

Note: Arrow symbols used with graphic number lines are not represented by the braille symbols shown above. Refer to the most recent edition of "Guidelines and Standards for Tactile Graphics" for techniques.

10.9 Arrows With Other Types of Arrowheads: Here are the blunted, straight, and curved arrowheads identified in the Nemeth Code. Examples with a single shaft of ordinary length are shown.

<u>Blunted Arrowheads</u>		(the longer line represents the arrow shaft)	
Full (left or right-pointing)			
			
			
Lower only (left- or right-pointing)			
			
Upper only (left- or right-pointing)			
			
<u>Straight Arrowheads</u>		(the longer line represents the arrow shaft)	
Full (left or right-pointing)			
			
			
Lower only (left- or right-pointing)			
			
Upper only (left- or right-pointing)			
			
<u>Curved Arrowheads</u>			
Full, left-pointing			
Full, right-pointing			
			
Lower only, left-pointing			
Lower only, right-pointing			
Upper only, left-pointing			
Upper only, right-pointing			

10.11 Curved Arrows: Direction indicators are not used with curved arrows. The direction of curvature is shown in braille by a left- or right-pointing arrowhead in the following manner. A curved shaft *preceded* by a *left*-pointing arrowhead represents a *clockwise* arrow. A curved shaft *followed* by a *right*-pointing arrowhead represents a *counterclockwise* arrow.

<u>Curved Shaft</u>	
Clockwise	
Counterclockwise	

PRACTICE 10C

Other Types of Arrow Shafts

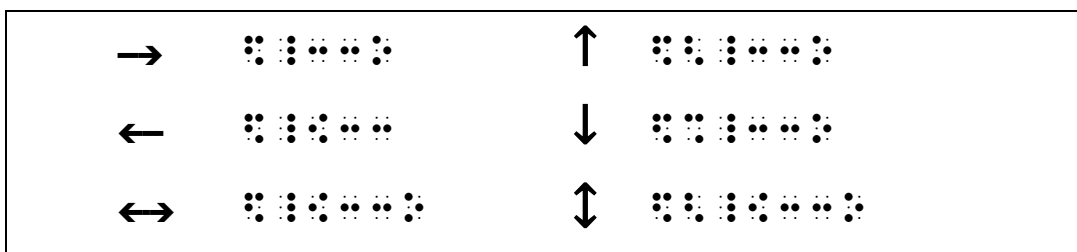
<u>Vertical</u>	<u>Slanted</u>	<u>Curved</u>

Can you figure out how to construct this spear with a northwest, blunted arrowhead?



Boldface and Compounded Arrows

10.12 Boldface Arrows: When an arrow is printed in boldface type and the typeform is determined to be significant and therefore retained, the boldface typeform indicator (dots 456) is incorporated into the structure of the arrow. The typeform indicator is placed at the beginning of the arrow symbol, before the arrowhead or arrow shaft. When a direction indicator is required, the direction indicator is brailled first, followed by the boldface typeform indicator.



Reminder: A right-pointing arrow in nonregular type requires a shaft. The contracted form is not used.

10.13 Arrows Used as Signs of Comparison Compounded Vertically: Lesson 6 introduced signs of comparison compounded vertically—two or more simple signs of comparison arranged one under the other in print. When arrows are so arranged, the same principle applies. The combination of symbols becomes a single comparison sign "compounded vertically". The uppermost arrow symbol is brailled first, immediately followed by and unspaced from the symbol for the lower arrow. Each arrow requires a shape indicator.

When the arrow is part of a sign of comparison compounded vertically, the uncontracted form of the right-pointing arrow shown below is used. The contracted form of the right-pointing arrow is never used in these constructions.



Pointing-Right Over Pointing-Left	↔	⠠⠠⠠⠠⠠⠠⠠⠠⠠
Pointing-Left Over Pointing-Right	↔	⠠⠠⠠⠠⠠⠠⠠⠠⠠
Long Pointing-Right Over Short Pointing-Left	↔	⠠⠠⠠⠠⠠⠠⠠⠠⠠
Short Pointing-Right Over Long Pointing-Left	↔	⠠⠠⠠⠠⠠⠠⠠⠠⠠
Pointing-Right With Upper Barb Only Over Pointing-Left With Lower Barb Only	↔	⠠⠠⠠⠠⠠⠠⠠⠠⠠
Pointing-Right Over Boldface Pointing-Left	↔	⠠⠠⠠⠠⠠⠠⠠⠠⠠
Boldface Pointing-Left Over Pointing-Right	↔	⠠⠠⠠⠠⠠⠠⠠⠠⠠
Boldface Pointing-Right Over Boldface Pointing-Left	↔	⠠⠠⠠⠠⠠⠠⠠⠠⠠

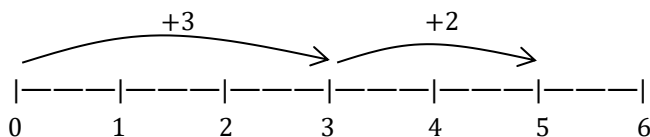
10.15 Other Arrows: It is recommended that Nemeth Code arrows not be used for nonmathematical purposes. Rules and guidelines in BANA sourcebooks examine flowchart arrows, arrows in graphic organizer diagrams, lead lines in tactile graphics, etc. Here are some samples to consider. (Transcriptions are not shown.)

Example 10.15-1

$$(2x + 3)(x - 5) = 2x^2 - 10x + 3x - 15$$

It may be better to present these arrows as a tactile graphic.

Example 10.15-2



Methods for depicting arrows above number lines are given in Guidelines and Standards for Tactile Graphics.

Example 10.15-3 $\frac{3}{4}$ ← fraction line

The lead line in this diagram could be omitted without a loss of information.

Refer to *Guidelines and Standards for Tactile Graphics* for techniques regarding arrows in specific applications such as box-and-whisker plots, clock faces, graphs, line plots, measurement tools, number lines, and spinners.

INTRODUCTION TO SPATIAL ARRANGEMENTS

10.16 Background: Up to this point we have looked at mathematical material that is read linearly, from left to right, whether it appears embedded within the narrative or set apart from the text as displayed material. When material is arranged on more than one line in print and there is essential vertical alignment of place values or characters, a spatial arrangement is required in braille. This lesson teaches rules for the transcription of spatially-arranged addition and subtraction problems. *Note: The term "alignment" refers to vertical location in the same column (cell).*

Spatial Arrangements with Addition and Subtraction

Lesson 1 introduced linear problems using the symbol + for "plus," – for "minus, and = for "equals".

$$2 + 3 = 5 \quad \dots \dots \dots \dots$$

$$7 - 6 = 1 \quad \dots \dots \dots \dots$$

When addition and subtraction problems are printed in a vertical arrangement, Nemeth braille rules for spatial arrangements apply. We will use standard terminology to refer to the parts.

2	<i>addend</i>	7	<i>minuend</i>
+ 3	<i>addend</i>	– 6	<i>subtrahend</i>
—	<i>separation line</i>	—	<i>separation line</i>
5	<i>sum</i>	1	<i>difference</i>

10.17 Numeric Indicator: The numeric indicator is generally not used in arrangements which are aligned for computation. Use of the numeric indicator in certain situations follows rules particular to the specific topic. This distinction will be clarified as each topic is presented.

10.18 Separation Line: The line printed between the addends and the sum or between the subtrahend and the difference (signifying "equals") is called the "separation line" and is brailled as a line of dots 25. The separation line must extend one cell to the left and to the right of the longest entry appearing above or below it.

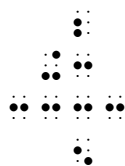
Separation Line (varying in length) ⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨

10.19 Alignment with Addition and Subtraction: In spatial arrangements for addition and subtraction, the corresponding digits, commas, and decimal points are vertically aligned one below the other. In other words, digits under digits, commas under commas, decimal points under decimal points.

Examples Placement of code switch indicators with spatial arrangements is discussed later in this lesson. Assume Nemeth context in all examples in this section. The switch indicators are not shown.

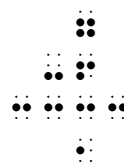
Example 10.19-1

$$\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array}$$



Example 10.19-2

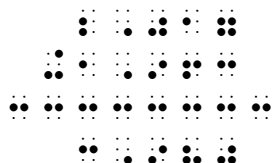
$$\begin{array}{r} 7 \\ -6 \\ \hline 1 \end{array}$$



Numeric indicators are omitted. Digits are aligned by place value, as printed. The separation line extends one cell to the left and one cell to the right of the widest row in the arrangement.

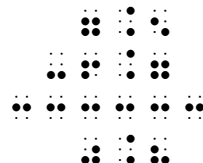
Example 10.19-3

$$\begin{array}{r} 2,017 \\ +1,963 \\ \hline 3,980 \end{array}$$



Example 10.19-4

$$\begin{array}{r} 7.5 \\ -6.7 \\ \hline 0.8 \end{array}$$

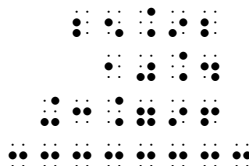


Commas and decimal points are aligned as printed. The separation line extends one cell to the left and one cell to the right of the widest row in the arrangement.

If items have been intentionally misaligned as an exercise for the student, the misalignment is preserved in the transcription. In the next example, the student has been instructed to arrange the digits for proper place value.

Example 10.19-5

$$\begin{array}{r} 25.92 \\ 10.4 \\ +3.796 \\ \hline \end{array}$$

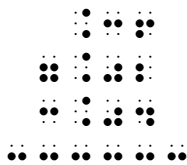


The decimal points are intentionally misaligned. The braille transcription matches the print arrangement.

The plus sign is not always present in addition problems. If there is no operation sign, examine the surrounding text to determine that this is indeed an addition problem. Then apply alignment rules for addition.

Example 10.19-6

$$\begin{array}{r} .36 \\ 7.02 \\ \hline 3.04 \end{array}$$

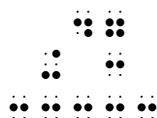


10.20 Placement of Symbols

10.20.1 Operation Symbols: Looking at the portion of the arrangement that lies above the separation line, the plus or minus symbol indicating addition or subtraction is placed at least one column of cells to the left of the widest column of numeric characters, regardless of print layout.

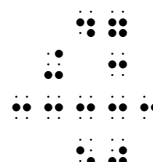
Example 10.20-1

$$\begin{array}{r} 47 \\ +3 \\ \hline \end{array}$$



Example 10.20-2

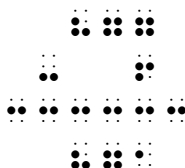
$$\begin{array}{r} 47 \\ +3 \\ \hline 50 \end{array}$$



The plus sign is printed directly to the left of the addend ("3"). In braille, the plus sign is placed one column to the left of the leftmost numeric character that appears above the separation line.

Example 10.20-3

$$\begin{array}{r} 877 \\ - 6 \\ \hline 871 \end{array}$$

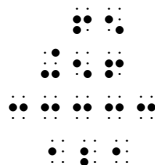


The minus sign is printed under the "8" in the minuend. In braille, the minus sign is placed one column to the left of the leftmost numeric character that appears above the separation line.

Placement of the operation symbol is in relation to the numeric characters above the separation line. If an answer is printed below the separation line, part of it may fall beneath symbols which appear above the separation line.

Example 10.20-4

$$\begin{array}{r} 65 \\ +56 \\ \hline 121 \end{array}$$



Place values "ones" and "tens" in the sum are aligned with those place values in the addends. The "hundreds" place value in the sum falls in the same column as the plus sign.

10.20.2 Currency Symbols: Currency symbols which appear above the separation line must also be similarly placed—that is, at least one column of cells to the left of the widest column of numeric characters which appears in the part of the arrangement above the separation line.

Example 10.20-5

\$.36	⠠⠠⠠⠠⠠⠠⠠⠠
7.02	⠠⠠⠠⠠⠠⠠⠠⠠
<u>+3.04</u>	⠠⠠⠠⠠⠠⠠⠠⠠
	⠠⠠⠠⠠⠠⠠⠠⠠

Placement of the currency symbol applies to the numeric characters above the separation line. If an answer is printed below the separation line, part of it may be shown beneath the symbols which appear above it.

Example 10.20-6

\$.36	⠠⠠⠠⠠⠠⠠⠠⠠
7.02	⠠⠠⠠⠠⠠⠠⠠⠠
<u>+3.04</u>	⠠⠠⠠⠠⠠⠠⠠⠠
\$10.42	⠠⠠⠠⠠⠠⠠⠠⠠

The decimal point and place values in the sum align with the decimal points and place values in the addends. The dollar signs above and below the separation line are not aligned in print and so need not be aligned in braille.

A symbol of operation or a currency symbol may be placed in the same position shown in print as long as the symbols above the separation line are placed at least one column of cells to the left of the widest column of numeric characters appearing there.

Example 10.20-7

\$1.45	⠠⠠⠠⠠⠠⠠⠠⠠
<u>- 1.05</u>	⠠⠠⠠⠠⠠⠠⠠⠠
\$.40	⠠⠠⠠⠠⠠⠠⠠⠠

In print, the minus sign appears to the left of the dollar sign. This alignment is duplicated in braille.

Example 10.20-8

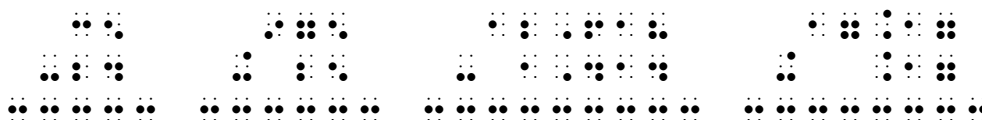
\$1.45	⠠⠠⠠⠠⠠⠠⠠⠠
<u>-1.05</u>	⠠⠠⠠⠠⠠⠠⠠⠠
\$.40	⠠⠠⠠⠠⠠⠠⠠⠠

In print, the minus sign appears directly beneath the dollar sign in print and the two dollar signs are vertically aligned. This alignment is duplicated in braille.

10.21 Side-by-Side Layout: Spatial arrangements may be placed side by side across a page. One blank space is left between the end of one separation line and the beginning of the next. This assures that at least three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement (separation lines excluded).

Example 10.21-1

35	975	12,618	17.17
<u>-24</u>	<u>+25</u>	<u>- 1,414</u>	<u>+ .17</u>

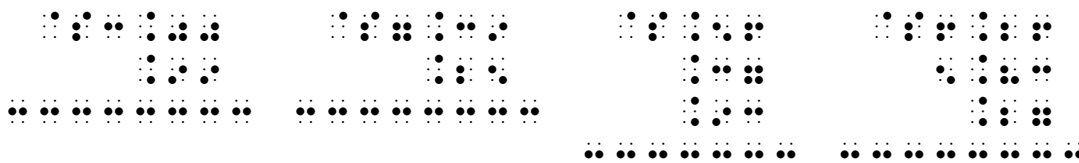


Each separation line extends one cell to the left and one cell to the right of the widest row in its arrangement. One clear column of blank space is between the ends of the separation lines.

Even if the separation lines are not on the same braille line, at least one clear column of blank space (one cell's width) must be between the ends of the separation lines.

Example 10.21-2

\$3.00	\$7.39	\$.56	\$6.26
<u>.99</u>	<u>.25</u>	.37	5.83
		<u>.93</u>	<u>.27</u>

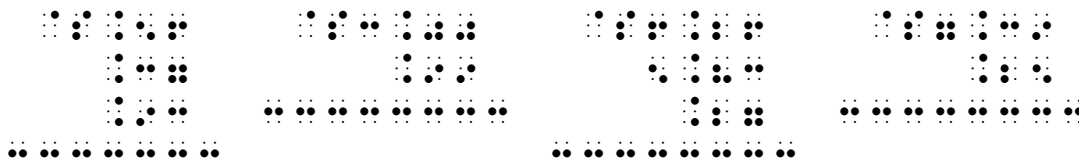


One clear column of blank space is between the ends of the separation lines, even those that do not fall on the same braille line.

The first addend in each problem is brailled on the same line, regardless of the print arrangement.

Example 10.21-3

\$.56	\$3.00	\$6.26	\$7.39
.37	<u>.99</u>	5.83	
<u>.93</u>		<u>.27</u>	<u>.25</u>



In print, the problems are arranged so that all separation lines are across the same row.

10.21.1 Page Number Restriction: If any portion of a problem falls on lines 1, 2, 24, or 25 of the braille page, at least three blank spaces must be left between the symbol furthest to the right of the overall arrangement including any separation lines and the first symbol of the page number. If this spacing cannot be achieved, the arrangement must end on line 23 or begin on line 3, respectively. An example is given in [10.32](#).

10.22 Blank Lines: A blank line is required above and below a spatial arrangement. Practice 10E demonstrates.

Instructions: Braille an opening Nemeth Code indicator on line 1 (cell 1). Line 2 should be blank. Begin the problems on line 3. Use side-by-side layout. Insert one blank line between the two sets of problems. After completion of the practice, leave one line blank and braille a Nemeth Code terminator in cell 1.

PRACTICE 10E

900 <u>+100</u> 1000	4391 <u>+ 81</u> 4472	25.763 <u>-4.239</u> 21.524	718 <u>-437</u> 281	5,025 <u>+3,999</u> 9,024
\$.27 6.13 <u>2.15</u>	\$3,854 602 <u>+ 918</u>	\$14.99 <u>- 6.37</u> \$ 8.62		

10.23 Omissions in Work Arranged Spatially for Computation: When a spatial arrangement contains omissions, in order to maintain necessary alignment *only the general omission symbol is used* regardless of how the omission is denoted in print. The number of general omission symbols to be used must be the same as the number of omission signs which occur in print.

General Omission Symbol ⠠⠠⠠

<i>Example 10.23-1</i>	946 <u>+??</u> 1002		<i>In print, the omissions are indicated as two questions marks.</i>
------------------------	---------------------------	--	--

<i>Example 10.23-2</i>	\$7.18 <u>-1.98</u> \$5.____	<i>Example 10.23-3</i>	\$7.18 <u>- ...</u> \$5.20
------------------------	------------------------------------	------------------------	----------------------------------

--	--

A long dash or an ellipsis does not give a clue regarding the number of characters that are omitted, therefore only one general omission symbol is brailled, aligned to the right.

10.24 Polynomials: In polynomials arranged spatially for addition or subtraction, terms and symbols are aligned.

Example 10.24-1

$$\begin{array}{r} 5r+16s+17t \\ -4r- 3s \\ \hline r+13s+17t \end{array}$$

If terms and symbols are not aligned in print, the braille transcription must show proper alignment unless the terms are intentionally misaligned in an exercise or illustration.

Example 10.24-2

$$\begin{array}{r} 4x+20y+6z \\ 18x-9z \\ \hline 5y+4z \end{array}$$

Terms are not aligned in this print example.

Example 10.24-3

$$\begin{array}{r} 2x-y-5z+9 = 0 \\ 7y-5z+28 = 0 \\ \hline 5y-11z-43 = 0 \end{array}$$

The separation line printed below these three equations characterizes this arrangement as an addition problem. Terms and symbols are aligned in braille even though, in this example, they are not aligned in print.

Superscripts, subscripts, and baseline indicators must align.

Example 10.24-4

$$\begin{array}{r} 3x^2+4xy+ 2 \\ x^2+2xy-15 \\ \hline 4x^2+6xy-13 \end{array}$$

All indicators align, as well as coefficients, and variables.

A space used for alignment does not cancel the need for a baseline indicator. When the baseline indicator is required, it is placed in the first (leftmost) possible position consistent with this alignment. Any spacing required for alignment is placed following the baseline indicator.

Example 10.24-5

$$\begin{array}{r} 3x^2 \quad +2 \\ \underline{x^2+4xy-5} \\ 4x^2+4xy-3 \end{array}$$

10.25 Abbreviations: When abbreviations occur in a spatially arranged addition or subtraction problem, they are vertically aligned and transcribed according to the rules for abbreviations.

Example 10.25-1

$$\begin{array}{r} 4 \text{ ft.} \quad 7 \text{ in.} \\ + 3 \text{ ft.} \quad 5 \text{ in.} \\ \hline 7 \text{ ft.} \quad 12 \text{ in.} \end{array}$$

Example 10.25-2

$$\begin{array}{r} 9.5\text{m} \quad 4\text{cm} \\ - 6 \quad \text{m} \quad 1\text{cm} \\ \hline \end{array}$$

A space is left between an abbreviation and the numeral to which it applies, even if the print copy shows no space.

10.26 Fractions: In a spatial addition or subtraction arrangement containing fractions, the fraction lines and the corresponding parts of fraction indicators are vertically aligned.

Example 10.26-1

$$\begin{array}{r} \frac{3}{4} \\ + \frac{1}{2} \\ \hline \end{array}$$

Example 10.26-2

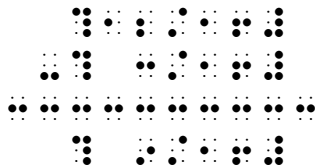
$$\begin{array}{r} \frac{1}{2} \\ + \frac{3}{4} \\ \hline \end{array}$$

These fractions are printed in regular type and on the baseline of writing, therefore fraction indicators are not used in braille.

Each numerator must be right justified in the column reserved for numerators, and each denominator must be left justified in the column reserved for denominators. Fraction indicators must also be vertically aligned. Blank cells may need to be inserted in order to achieve alignment.

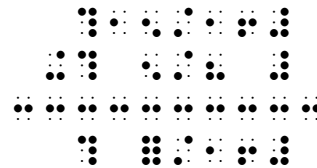
Example 10.26-3

$$\begin{array}{r} \frac{12}{16} \\ - \frac{3}{16} \\ \hline \frac{9}{16} \end{array}$$



Example 10.26-4

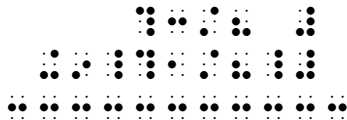
$$\begin{array}{r} \frac{15}{16} \\ + \frac{5}{8} \\ \hline \frac{?}{16} \end{array}$$



Fraction indicators must be right-justified in the columns reserved for both opening and closing indicators.

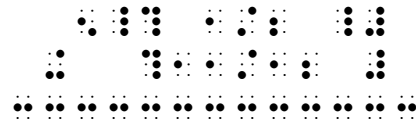
Example 10.26-5

$$\begin{array}{r} \frac{3}{8} \\ + 9\frac{1}{8} \\ \hline \end{array}$$



Example 10.26-6

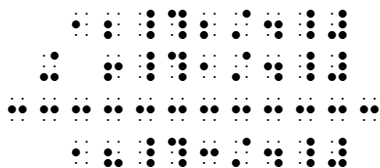
$$\begin{array}{r} 5\frac{1}{2} \\ + \frac{11}{12} \\ \hline \end{array}$$



The whole number parts of mixed numbers are vertically aligned by place value.

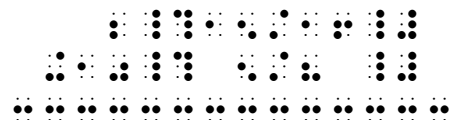
Example 10.26-7

$$\begin{array}{r} 12\frac{2}{4} \\ + 6\frac{1}{4} \\ \hline 18\frac{3}{4} \end{array}$$



Example 10.26-8

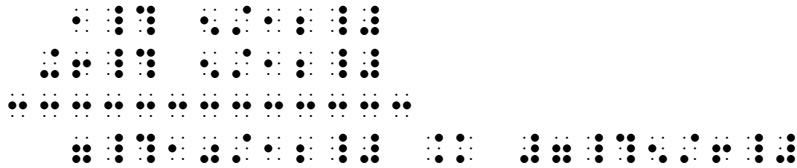
$$\begin{array}{r} 2\frac{15}{16} \\ + 10\frac{5}{8} \\ \hline \end{array}$$



The final answer in the next problem is not part of the vertical computation. Rules for linear expressions are followed – a numeric indicator is required.

Example 10.26-9

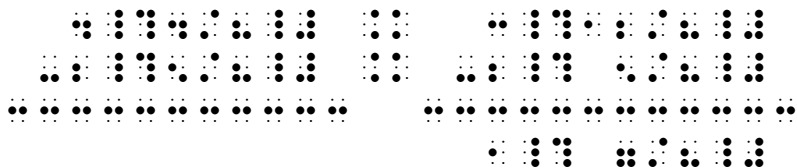
$$\begin{array}{r}
 1\frac{5}{12} \\
 + 6\frac{5}{12} \\
 \hline
 7\frac{10}{12} = 7\frac{5}{6}
 \end{array}$$



The equals signs in the next problem show a relationship between the two spatial arrangements.

Example 10.26-10

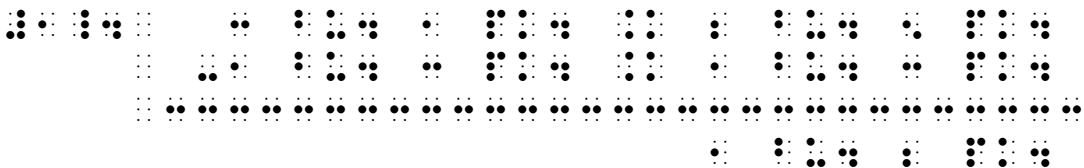
$$\begin{array}{r}
 4\frac{4}{8} = 3\frac{12}{8} \\
 - 2\frac{5}{8} = -2\frac{5}{8} \\
 \hline
 1\frac{7}{8}
 \end{array}$$



10.27 Placement of Identifiers: The numeric or alphabetic identifier associated with a spatially arranged addition or subtraction problem is positioned at the top line of the problem—that is, with the first addend or with the subtrahend. One clear column of blank space (one cell's width) must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement including separation lines.

Example 10.27-1

$$\begin{array}{r}
 \mathbf{1.} \quad 3 \text{ bu. } 1 \text{ pk.} = 2 \text{ bu. } 5 \text{ pk.} \\
 -1 \text{ bu. } 3 \text{ pk.} = 1 \text{ bu. } 3 \text{ pk.} \\
 \hline
 1 \text{ bu. } 2 \text{ pk.}
 \end{array}$$



10.27.1 Side-by-Side Arrangement: Spatially arranged itemized material may be arranged side-by-side. The identifiers are located on the same braille line across the width of the page, in sequential order. Problems are spaced so that no less than three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement (separation lines excluded) including the neighboring identifier.

Note that this is a special rule pertaining to spatially-arranged itemized material. As taught in **Lesson 3**, each main item in *nonspatial* itemized material must start in cell 1, even if the print copy arranges the items side-by-side across the page.

Example 10.27-2

$$\begin{array}{r}
 1) \quad 15 \\
 \quad \quad \underline{+3}
 \end{array}
 \quad
 \begin{array}{r}
 2) \quad 15 \\
 \quad \quad \underline{-3}
 \end{array}
 \quad
 \begin{array}{r}
 3) \quad 15 \\
 \quad \quad \underline{+13}
 \end{array}
 \quad
 \begin{array}{r}
 4) \quad 15 \\
 \quad \quad \underline{-13}
 \end{array}
 \quad
 \begin{array}{r}
 5) \quad 150 \\
 \quad \quad \underline{+130}
 \end{array}
 \quad
 \begin{array}{r}
 6) \quad 150 \\
 \quad \quad \underline{-130}
 \end{array}$$

A blank line precedes the second group of spatially-arranged problems (line 4, above). Spacing within and between problems is applied without regard to the arrangements above or below – that is, no attempt is made to align identifiers.

10.27.2 Page Number Restriction: If any portion of a problem falls on lines 1, 2, 24, or 25 of the braille page, at least three blank spaces must be left between the symbol furthest to the right of the overall arrangement including any separation lines and the first symbol of the page number. If this spacing cannot be achieved, the arrangement must end on line 23 or begin on line 3, respectively. An example is given in [10.32](#).

Instructions: Use side-by-side layout.

PRACTICE 10F

$$\begin{array}{r}
 1) \quad 621 \\
 + ??? \\
 \hline
 1096
 \end{array}
 \quad
 \begin{array}{r}
 2) \quad 17x - 8y - z \\
 - 2x + 17y + 6z \\
 \hline
 15x + 9y + \underline{\quad}
 \end{array}
 \quad
 \begin{array}{r}
 3) \quad 3x^2 - 5x + 4 \\
 - 5x^2 + 12x - 12 \\
 \hline
 -2x^2 + 7x - 8
 \end{array}$$

$$\begin{array}{r}
 4) \quad 1 \frac{2}{3} \text{ yr} \\
 + 5 \frac{5}{12} \text{ yr} \\
 \hline
 6 \frac{13}{12} \text{ yr} = 7 \frac{1}{12} \text{ yr}
 \end{array}$$

10.28 Carried Numbers with Addition: Carried numbers should appear in the same columnar position as in print. A line of carried number indicators the same length as the separation line is inserted between the carried numbers and the first line of the addition.

Carried Number Indicator (varying in length) ⠠⠠⠠⠠⠠⠠

Example 10.28-1

	11	⠠⠠
	213	⠠⠠⠠⠠⠠⠠
	+ 87	⠠⠠⠠
	<u>300</u>	⠠⠠⠠⠠⠠
		⠠⠠⠠⠠

In the print copy, the carried numbers are in a smaller typeface and a carried number line is not present.

10.28.1 Placement of Identifiers with Carried Numbers: An identifier, if present, is placed on the first line of the addition problem (the first addend) regardless of the presence of carried numbers. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 10.28-2

	21	⠠⠠
1.	25.9	⠠⠠⠠⠠⠠⠠⠠
	49.6	⠠⠠⠠⠠⠠
	<u>87.2</u>	⠠⠠⠠⠠⠠
	162.7	⠠⠠⠠⠠⠠⠠⠠
		⠠⠠⠠⠠⠠

Introduction to Cancellation

A spatial arrangement is required whenever numbers, letters, abbreviations, or words in a mathematical expression are canceled in print by any type of stroke through them. The opening and closing cancellation indicators enclose the material being canceled.

Opening Cancellation Indicator	⦿
Closing Cancellation Indicator	⦿

⦿ ⦿	⦿ ⦿ ⦿
⦿ h	⦿ ⦿ ⦿
⦿ ft	⦿ ⦿ ⦿ ⦿ ⦿

10.29 Cancellation in Subtraction Problems: When cancellation is shown in subtraction, the material should be aligned for computation. Spaces are inserted where necessary to achieve this.

Example 10.29-1

4 10	⦿ ⦿	⦿ ⦿
1 7	⦿ ⦿ ⦿ ⦿	⦿ ⦿
- 39	⦿ ⦿	⦿
111	⦿ ⦿ ⦿ ⦿ ⦿ ⦿ ⦿ ⦿ ⦿ ⦿ ⦿ ⦿	⦿ ⦿
	⦿ ⦿	⦿

Place values align throughout the arrangement. Numerals do not appear in the same columns as cancellation indicators.

Example 10.29-2

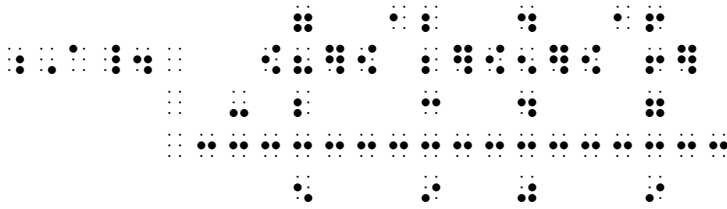
11	⦿ ⦿
7 1 13	⦿ ⦿ ⦿ ⦿
8 2 3	⦿ ⦿ ⦿ ⦿
- 2 5 4	⦿ ⦿
5 6 9	⦿ ⦿ ⦿ ⦿ ⦿ ⦿ ⦿ ⦿ ⦿ ⦿ ⦿ ⦿
	⦿ ⦿

Cancellation indicators align with cancellation indicators. The minus sign (line 4) is placed one column to the left of the opening cancellation indicator on line 3.

10.29.1 Placement of Identifiers with Spatial Subtraction: An identifier, if present, is placed on the first line of the subtraction problem (the minuend) regardless of the presence of canceled items. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 10.29-3 **A.**

$$\begin{array}{r}
 7 12 4 16 \\
 \cancel{8} \cancel{2} \cancel{5} \cancel{6} \\
 \hline
 2 3 4 7 \\
 \hline
 5 9 0 9
 \end{array}$$



PRACTICE 10G

Carried Numbers and Cancellation

1.

$$\begin{array}{r}
 11 \\
 948 \\
 + 75 \\
 \hline
 1023
 \end{array}$$

2.

$$\begin{array}{r}
 12 2 \\
 \$ 2.52 \\
 65.89 \\
 91.99 \\
 \hline
 32.03 \\
 \hline
 \$192.43
 \end{array}$$

3.

$$\begin{array}{r}
 4 16 1 13 \\
 \cancel{5} \cancel{6} \cancel{2} \cancel{3} \\
 \hline
 8 0 4 \\
 \hline
 4 8 1 9
 \end{array}$$

Arrangement on the Page

A spatial arrangement should be confined to one braille page. Strategies for larger arrangements are beyond the scope of this course.

10.30 Blank Lines and the Page Change Indicator: A blank line is required above and below a spatial arrangement. The presence of a page change indicator does not affect this requirement. There may be times when a blank line is needed both before and after the page change indicator; for example, when spatial arrangements occur before and after the page change, or when a heading starts the new print page.

Example 10.30-1

15	15
<u>+13</u>	<u>-13</u>

===== new print page 123

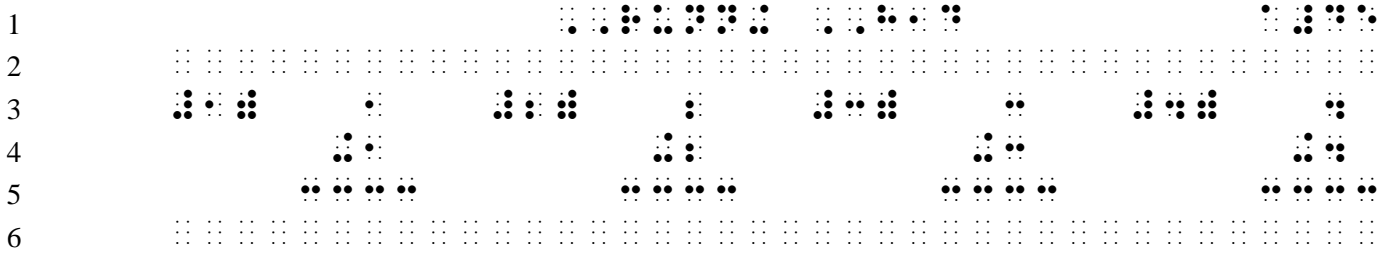
150	150
<u>+130</u>	<u>-130</u>

The image shows the Braille representation of the arithmetic problems. Each problem is presented in a 4x4 grid of dots. The first problem shows 15 + 13 = 28 and 15 - 13 = 2. The second problem shows 150 + 130 = 280 and 150 - 130 = 20. There are blank lines before and after each problem, and a page change indicator between the two problems.

Each row of spatial problems requires a blank line before and after it.

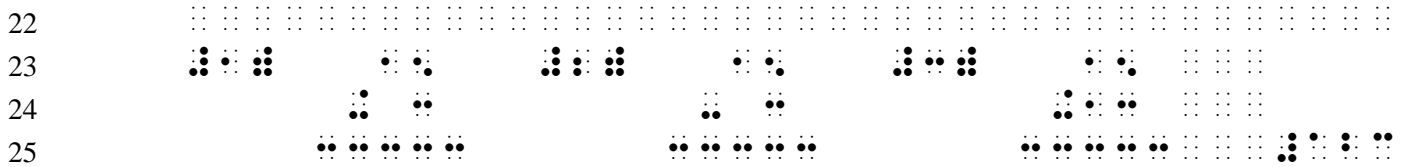
If a running head is in use, line 2 must be blank, beginning the spatial arrangement on line 3.

Example 10.31-2



10.31.2 Ending a Braille Page with a Spatial Arrangement: A spatial arrangement may be placed on the last (bottom) line of the braille page as long as there is room for at least three blank spaces between the symbol furthest to the right of the overall arrangement *including any separation lines* and the first symbol of the braille page number.

Example 10.31-3



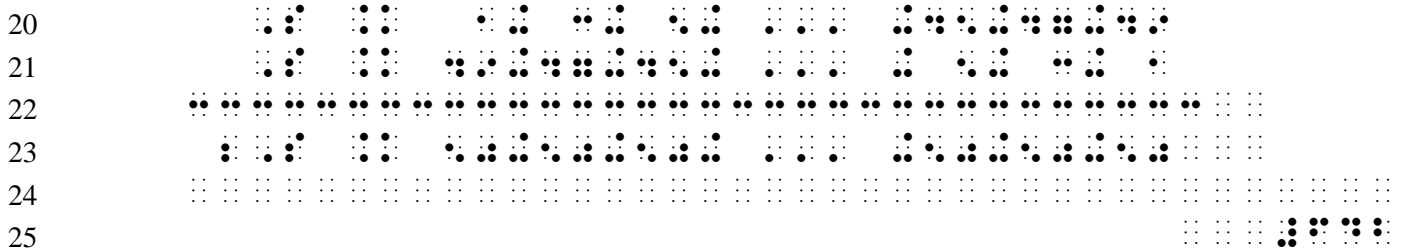
See the restriction regarding wide arrangements, below.

10.32 Wide Arrangements: When a spatial arrangement ends on the 24th or 25th line of a braille page or begins on the 1st or 2nd line of a braille page, at least three blank spaces must be left between the symbol furthest to the right of the overall arrangement *including any separation lines* and the first symbol of the page number. If this spacing cannot be achieved, the arrangement must end on line 23 or begin on line 3, respectively.

Example 10.32-1

$$\begin{aligned}
 S &= 1+ 3+ 5+ \dots +45+47+49 \\
 S &= 49+47+45+ \dots + 5+ 3+ 1 \\
 \hline
 2S &= 50+50+50+ \dots +50+50+50
 \end{aligned}$$

... at the bottom of a braille page



... at the top of a braille page (with no running head)

1
2
3
4
5
6

10.33 Itemized Spatial Problems with Subdivisions: When a set of spatial arrangements contains both main divisions and subdivisions, the following must be observed. If there is no material between the main division identifier and the first subdivision identifier, place the main division at the margin in cell 1, and place the first subdivision on the same braille line after one blank cell. As many subdivisions as possible may be placed across the line. The identifiers are placed as outlined in [10.27](#), [10.28.1](#), and [10.29.1](#), and must be located on the same braille line across the width of the page.

Example 10.33-1

1. a. $\begin{array}{r} 15 \\ +3 \\ \hline \end{array}$ b. $\begin{array}{r} 15 \\ -3 \\ \hline \end{array}$

If additional subdivisions remain after the width of the page has been met, insert a blank line below the last line of the arrangements. Then begin the next subdivision identifier in cell 3.

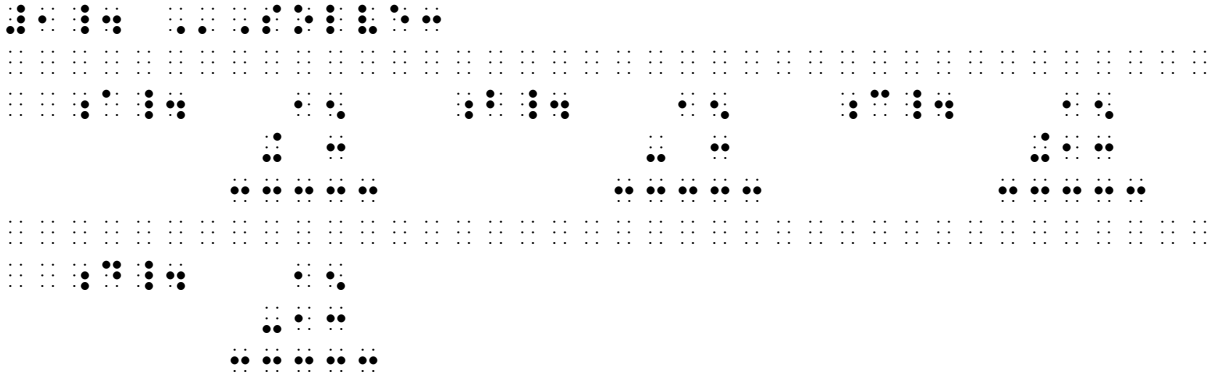
Example 10.33-2

1. a. $\begin{array}{r} 15 \\ +3 \\ \hline \end{array}$ b. $\begin{array}{r} 15 \\ -3 \\ \hline \end{array}$ c. $\begin{array}{r} 15 \\ +13 \\ \hline \end{array}$ d. $\begin{array}{r} 15 \\ -13 \\ \hline \end{array}$

If there is material associated with the main division number, the subdivisions begin on the next available line (following the required blank line that precedes spatial material) starting in cell 3.

Example 10.33-3

1. Solve: a. $\begin{array}{r} 15 \\ +3 \\ \hline \end{array}$ b. $\begin{array}{r} 15 \\ -3 \\ \hline \end{array}$ c. $\begin{array}{r} 15 \\ +13 \\ \hline \end{array}$ d. $\begin{array}{r} 15 \\ -13 \\ \hline \end{array}$



Placement of Code Switch Indicators

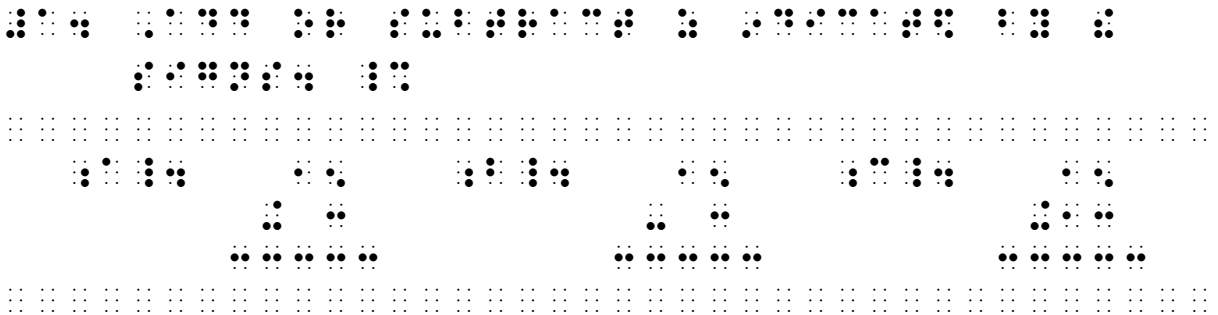
To keep the code switch indicators from interfering with the layout of the problem, they are placed outside of the arrangement. Position the required blank lines *within* the Nemeth Code switches as described below.

10.34 Opening Nemeth Code Indicator: If space permits, an opening Nemeth Code indicator that precedes a spatial problem may be placed on the same line with the end of the text above the problem.

Example 10.34-1

1. Add or subtract as indicated by the signs.

a. $\begin{array}{r} 15 \\ +3 \\ \hline \end{array}$ b. $\begin{array}{r} 15 \\ -3 \\ \hline \end{array}$ c. $\begin{array}{r} 15 \\ +13 \\ \hline \end{array}$



The opening Nemeth Code indicator is placed outside of the arrangement, before the required blank line.

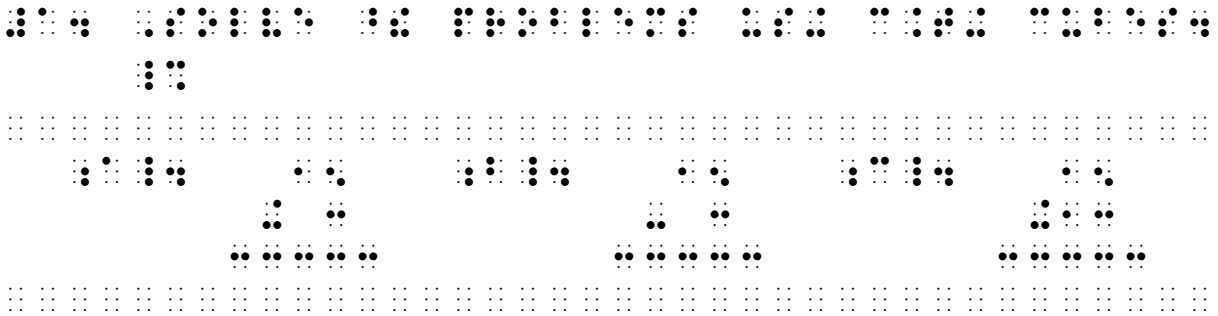
The BANA Nemeth Code Technical Committee is discussing details regarding placement of the opening Nemeth Code indicator preceding itemized spatial arrangements when the indicator does not fit on the same line as the preceding text. The examples below show the opening switch in the runover position of the current paragraph.

If there is not room on the line with the preceding text, the opening Nemeth Code indicator is placed in the runover position. The opening switch indicator will be by itself on that line. The required blank line follows. The switch indicator does not take the place of the required blank line preceding the spatial arrangement.

Example 10.34-2

1. Solve these problems using counting cubes.

a.
$$\begin{array}{r} 15 \\ +3 \\ \hline \end{array}$$
 b.
$$\begin{array}{r} 15 \\ -3 \\ \hline \end{array}$$
 c.
$$\begin{array}{r} 15 \\ +13 \\ \hline \end{array}$$



10.35 Nemeth Code Terminator: When Nemeth Code is terminated after a spatial problem, first insert the required blank line that follows the arrangement. Then place the Nemeth Code terminator in cell 1 by itself on the following braille line. The code switch indicator does not take the place of the required blank line following the spatial arrangement. Resume UEB text on the next line.

Example 10.35-1

1. Solve these problems using counting cubes.

a.
$$\begin{array}{r} 15 \\ +3 \\ \hline \end{array}$$
 b.
$$\begin{array}{r} 15 \\ -3 \\ \hline \end{array}$$
 c.
$$\begin{array}{r} 15 \\ +13 \\ \hline \end{array}$$

2. Solve these problems with your pencil.

